

**DEVELOPMENT OF SPECIALIZED BASE PRIMITIVES FOR
MESO-SCALE CONFORMING TRUSS STRUCTURES**

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**DEVELOPMENT OF SPECIALIZED BASE PRIMITIVES FOR
MESO-SCALE CONFORMING TRUSS STRUCTURES**

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For Carissa, who helped celebrate the leaps, mourn the falls,
and gave me a good kick when I needed one.

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	iv
LIST OF TABLES	vii
LIST OF FIGURES	viii
LIST OF SYMBOLS AND ABBREVIATIONS	ix
SUMMARY	x
CHAPTER 1 : BACKGROUND AND MOTIVATION.....	1
1.1 : BACKGROUND	1
1.2 : MOTIVATION	4
1.3 : RESEARCH QUESTIONS AND HYPOTHESES	6
1.4 : VALIDATION OF THE DESIGN METHOD	8
1.5 : ORGANIZATION OF THE THESIS	12
CHAPTER 2 : LITERATURE REVIEW AND GAP ANALYSIS	15
2.1 : LITERATURE REVIEW	15
2.1.1 : Cellular Structures	15
2.1.2 : Lattice Structure Analysis.....	17
2.1.3 : Optimal Lattice Structures	19
2.2 : GAP ANALYSIS	25
2.3 : SUMMARY	26
CHAPTER 3 : THE UNIT-CELL LIBRARY APPROACH.....	28
3.1 : THE UNIT-CELL APPROACH	29
3.2 : CURRENT OPTIMIZATION APPROACH	31
3.3 : UNIT-CELL LIBRARY CONCEPT	33
3.4 : SPECIALIZED UNIT-CELL CREATION	34
3.5 : THE UNIT-CELL LIBRARY	37
3.6 : SUMMARY	40
CHAPTER 4 : THE DESIGN PROCESS	42
4.1 : MOTIVATION AND CONCEPT	43
4.2 : PROBLEM DEFINITION	46
4.3 : SOLID BODY ANALYSIS.....	47
4.4 : UNIT-CELL MODEL	49
4.5 : MODEL CORRELATION	51
4.6 : TOPOLOGY GENERATION	52
4.6.1 : Setting Minimum, Maximum, and Cutoff Diameters.....	54
4.7 : AMBIGUITY RESOLUTION.....	60
4.8 : FINAL STRUCTURE	61
4.9 : SUMMARY	61
CHAPTER 5 : EXAMPLE PROBLEMS	63

5.1 : A SIMPLE EXAMPLE PROBLEM.....	64
5.1.1 : Identically Sized Lattice Structure.....	66
5.1.2 : Particle Swarm Optimization.....	67
5.1.3 : Levenburg-Marquardt/Least Squares Minimization.....	71
5.1.4 : Unit-Cell Library Approach.....	73
5.1.5 : Summary of the First Example Problem.....	77
5.2 : SECOND EXAMPLE PROBLEM.....	79
5.2.1 : Identically Sized Truss Structure.....	80
5.2.2 : PSO Optimization.....	81
5.2.3 : Levenburg-Marquardt/Least Squares Minimization.....	84
5.2.4 : Unit-Cell Library and Solid-Body Analysis.....	88
5.2.5 : Summary of Second Example Problem.....	90
5.3 : EXAMPLE PROBLEM THREE – A COMPLEX PROBLEM.....	91
5.3.1 : Discussion on Optimization.....	93
5.3.2 : Solid-Body Analysis and Unit-Cell Library Method.....	95
5.3.3 : Summary of Third Example Problem.....	96
5.4 : SUMMARY.....	97
CHAPTER 6 : SUMMARY AND CONCLUSIONS	99
6.1 : CONCLUSIONS.....	102
6.1.1 : Specialized Unit-cells.....	103
6.1.2 : Strut-Level Optimization.....	104
6.1.3 : Solid Body Analysis.....	104
6.1.4 : Performance.....	105
6.1.5 : Conclusions.....	105
6.2 : CONTRIBUTIONS.....	106
6.2.1 : Unit-Cell Library.....	107
6.2.2 : Optimization-Free Design.....	108
6.3 : FUTURE WORK.....	108
6.3.1 : Streamlining.....	108
6.3.2 : Solid Body/Unit-Cell Library Correlation.....	109
6.4 : CLOSURE.....	110
REFERENCES	111

LIST OF TABLES

Table 1-1: Hypotheses Verification Outline	11
Table 3-1: The Unit-Cell Library.....	38
Table 4-1: Complexity of Different Lattice Design Approaches.....	45
Table 4-2: Unit-Cell Model Definition	51
Table 4-3: Maximum and minimum diameters for preferred lattice structures.....	57
Table 5-1: Parameters for particle swarm optimization of first example problem	68
Table 5-2: Results of PSO for the First Example	69
Table 5-3: Parameters for least squares minimization of the first example problem	71
Table 5-4: Averaging and scaling of unit-cells.....	75
Table 5-5: Summary of Cantilever Beam Example Results	78
Table 5-6: Comparison of Starting Configurations	81
Table 5-7: Particle Swarm Optimization Parameters for Example Two	82
Table 5-8: PSO results for the second example problem.....	84
Table 5-9: Parameters for least squares minimization of the first example problem	85
Table 5-10: LM/LSM results for the second example problem.....	86
Table 5-11: Results of Lattice Structure Created Using Solid-Body Analysis.....	90
Table 5-12: Summary of Results for the Second Example Problem	91
Table 5-13: Summary of Results from Third Example Problem.....	97
Table 6-1: Hypotheses Verification Outline	103

LIST OF FIGURES

Figure 1-1: A robotic gripper manufactured by Stratasys	3
Figure 1-2: Organization of the thesis	12
Figure 2-1: Two examples of designed lattice structures	16
Figure 2-2: Octet-truss unit cell	18
Figure 2-3: One of Michell's 1904 solutions, from [27]	19
Figure 3-1: Unit-Cell Definition of a Model.....	30
Figure 3-2: Unit cells optimized for six loading conditions	36
Figure 4-1: Unit-Cell Library Approach.....	46
Figure 4-2: The Problem Definition	47
Figure 4-3: Solid body analysis	48
Figure 4-4: Stress information from finite-element analysis	49
Figure 4-5: Representation of model defined as a set of empty unit-cells.....	50
Figure 4-6: Resulting model showing sized lattice struts	54
Figure 4-7: Study of the effects of minimum strut diameter on displacement	56
Figure 4-8: Study of the effects of maximum diameter when volume is held constant ...	56
Figure 4-9: Effects of cutoff diameter on tip displacement	59
Figure 4-10: Effects of cutoff diameter on volume	59
Figure 5-1: First Example Problem.....	65
Figure 5-2: Base truss for cantilever beam example.....	65
Figure 5-3: Iteration history for PSO optimization of first example problem	70
Figure 5-4: First example lattice structure after PSO optimization	70
Figure 5-5: First example problem lattice structure derived using LM/LSM.....	72
Figure 5-6: Solid-body finite element analysis of the cantilever beam example.....	73
Figure 5-7: First example problem: unit-cell library and solid-body analysis.....	77
Figure 5-8: Identically Sized Triangular Truss Structure	80
Figure 5-9: Identically Sized Quadrilateral Truss Structure	81
Figure 5-10: Optimization History of PSO for the Second Example Problem.....	83
Figure 5-11: Example problem two after PSO optimization	84
Figure 5-12: Objective Function History for LM/LSM of Second Example.....	87
Figure 5-13: Result of LM/LSM of Second Example	87
Figure 5-14: Ansys Stress Distribution in Second Example.....	88
Figure 5-15: Truss Structure Specialized Using Solid-Body Analysis.....	88
Figure 5-16: Plot of the Effect of Maximum Diameter on Compliance	89
Figure 5-17: Saddle Skin Geometry	92
Figure 5-18: Saddle-Shaped, Identically Sized Lattice Structure	93
Figure 5-19: Ansys model of the Third Example Problem.....	95
Figure 5-20: Specialized Lattice Structure for Saddle-Shaped Example.....	96

LIST OF SYMBOLS AND ABBREVIATIONS

FDM.....	Fused Deposition Modeling
LSM	Least Squares Minimization
PSO	Particle Swarm Optimization
SLA.....	Stereolithography
SLM	Selective Laser Melting
SLS.....	Selective Laser Sintering
d	Displacement
D_c	Cutoff Diameter
D_i	Strut Diameters
$D_{i,norm}$	Normalized Strut Diameters
$D_{i,max}$	Maximum Strut Diameter
D_{max} / D_{min}	User Defined Diameter Range
J	Compliance
$N_{i,1}$ or $N_{i,2}$	Strut Nodes
ρ	Relative Density
ρ_e	Dummy Size Variable
P_d	Diameter Size Penalty
p	Cutoff Percent
$S_{i,j}^u$	Stress Scaling Factor
$S_{j,k}^L$	Unit-Cell Library Scaling Factor
U	Strain Energy
V	Volume
w_v	Volume Weighting Value
w_p	Diameter Size Penalty Weighting Value
w_U	Strain Energy Weighting Value

SUMMARY

The advent of rapid manufacturing has enabled the realization of countless products that have heretofore been infeasible. From customized clear braces to jet fighter ducts and one-off dental implants, rapid manufacturing allows for increased design complexity and decreased manufacturing costs. The manufacturing capabilities of this process have evolved to the point that they have surpassed current design capabilities. Meso-scale lattice structures can now be built that contain more lattice struts than it is reasonable to efficiently define. This work has attempted to create a method for designing such lattice structures that is efficient enough to allow for the design of large or complex problems.

The main hindrance to the design of complex meso-scale lattice problems is essentially the need to define the strut diameters. While it is obvious that a large design would contain more struts than can be specified by hand, designs also quickly surpass the current capabilities of computational optimization routines. To overcome this problem, a design method has been developed that uses a unit-cell library correlated to finite element analysis of the bounding geometry to tailor the structure to the anticipated loading conditions. The unit-cell library is a collection of base lattice primitives, or unit-cells, that have been specialized for certain applications. In this case, primitives have been created that perform best under the types of stress analyzed by finite element analysis.

The effectiveness of this process has been demonstrated through several example problems. In all cases, the unit-cell library approach was able to create structures in less time than current methods. The resulting structures had structural performance slightly

lower than similar models created through optimization methods, although the extent of this degradation was slight. The method developed in this work performs extremely well, and is able to create designs for even the most complex lattice structures. There is room for future development, however, in the streamlining of the design process and consideration of higher-order affects within unit-cells.

Chapter 1: Background and Motivation

The use of additive manufacturing, which creates final part geometry through the addition of material rather than removal of excess material, is gaining popularity for parts with high complexity or low volume production. The ability to create three dimensional components without the need for expensive fixtures or molds allows for economical production runs of only one or two parts. Additionally, highly complex parts can be produced that would be impractical or impossible using standard manufacturing methods. This trend allows for the realization of designs that previously would have been infeasible.

One of the products that, with the introduction of additive manufacturing, is now feasible is meso-scale lattice structures. These structures are similar to trusses, but have components in three dimensions instead of just two. They are considered meso-scale because their size falls between that of the geometry of the part (macro-scale) and the material properties (micro-scale). Methods have previously been developed to design meso-scale lattice structures such that they conform to a pre-existing geometrical shape. However, the difference in orders of magnitude between the lattice structure and its bounding geometry results in many lattice members (struts), many more than it is feasible to manually design. The design of such structures is the focus of this work.

1.1: Background

Additive manufacturing, or rapid manufacturing, has become increasingly popular with individually customized and low-volume components. The production of hearing

aids and dental implants, which both traditionally involve labor-intensive processes of molding and casting, has been replaced by selective laser sintering of plastics and metals. These processes create parts that are fully customized to scans made of the patient's body but do not require master patterns or molds, whose manufacture by traditional methods requires many hours of labor by highly skilled craftsmen. Similarly, the production of military jet fighters involves thousands of components that must simultaneously be complex and lightweight. Rapid manufacturing allows such non-structural items as air ducts to be designed as a single complex unit, rather than multiple parts that are less efficient but manufacturable with traditional methods [17]. This results in a part with less excess material and higher efficiency within its particular system.

Stereolithography (SLA) is a process widely utilized as a form of rapid prototyping, but also plays a part in rapid tooling and rapid manufacturing [18]. During the SLA process, a platform is incrementally lowered into a vat of light activated photopolymer. Between each platform movement, a laser scans a "slice" of the part being built onto the surface of the resin. As the platform lowers, a small amount of uncured resin is spread across the top of the cured layers, creating the next layer of the part. This process repeats until the entire part is built. It is possible to vary the material properties by using different resins that, developed during the many years of SLA's use as a prototyping tool, often mimic traditional manufacturing plastics such as ABS.

This process poses several advantages and disadvantages to the specific field of rapid manufacturing. Although material properties are often analogous to common plastics, a trait that proves useful for rapid prototyping, parts are subject to rapid aging and become brittle and discolored over time due to the nature of the polymer reaction

employed. While this has limited the manufacturing use of stereolithography to components requiring relatively little strength, great success has been realized in areas where the primary concern is complexity. As one of the first additive manufacturing processes, stereolithography has reached maturity, resulting in highly reliable and accurate machines. Hearing aid shells can be customized to a patient's ear simply by scanning the interior of their ear canal and using that data to create part geometry. A shell is then produced in stereolithography that fits perfectly [7]. Align Technologies, makers of the Invisalign® brand of clear braces, uses stereolithography to create molds for braces that are designed specifically for the needs of the individual [24]. For parts requiring more strength, selective laser sintering (SLS) or selective laser melting (SLM) of metals or plastics is often employed. As previously mentioned, SLS in gold is quickly becoming the most economical way to produce dental implants and crowns, and has successfully been implemented to create ductwork for jet fighters in other materials [36]. Fused deposition modeling (FDM), which extrudes a series of very thin beads of polymer to build part geometry, is also common in rapid manufacturing. One such example of an FDM part is the robotic gripper pictured below, which has suction channels built in to the arms of the gripper [36].



Figure 1-1: A robotic gripper manufactured by Stratasys

1.2: Motivation

When designing structural systems, it is often desirable to utilize materials that are both high in stiffness and low in weight. This need is most prevalent in the avionics industry, in which weight directly translates to not only material costs, but long-term operating costs as well. Stiffness/weight relationships are likely to become similarly prominent in the automobile industry as the public demands more efficient vehicles in response to increased gasoline prices. While material selection plays a large part in this compromise, the design of the structure itself also affects the stiffness characteristics. Unfortunately, the amount that structural designs can be altered to achieve desirable mechanical characteristics is often limited by the design requirements of the system. In response to this limitation, researchers have developed meso-scale lattice structures that allow tuning of material and mechanical characteristics with only limited changes in overall part geometry.

Meso-scale lattice structures are small features that act as reinforcement within a larger part. Lattice structures are generally considered to be meso-scale if their struts (or individual lattice members) are on the order of 1-10mm. They are considered a deterministic cellular structure, since each individual strut of the lattice can be individually specified and designed. Inclusion of meso-scale lattice structures in a structural design allows for additional tuning of the mechanical properties of a part beyond the micro-scale material properties and the macro-scale part geometry. Since the lattice structure is an order of magnitude smaller than the part in which it resides, significant changes can be made to the lattice without necessitating changes to the bounding geometry of the macro-scale component. The advent of rapid manufacturing

enables implementation of this meso-scale lattice, as it is one of the only cost-efficient methods to create such complex geometry. However, although the manufacturing of such complex parts poses little challenge, their design taxes the limits of both human and computerized design methods [35].

The design challenge posed by meso-scale lattice structures is that of complexity. Parts of reasonably modest size, 20x20cm for example, might have 4,000 to 8,000 individual lattice struts. If the struts are considered on an individual basis, this results in 4,000-8,000 individual design variables, considering only the diameters of each strut. If the lattice configurations must also be individually designed, the complexity of the problem further increases. The sheer number of variables, and their interaction to create many local minimums, makes optimization challenging [8]. Optimization methods such as genetic algorithms and particle swarm optimization have been developed specifically for such problems of large scale and complexity [35]. However, since the complexity of a design problem is related exponentially to the number of design variables, the practicality of such methods is limited to designs that fall below the level of “modest.” Beyond the realm of reasonably small designs, the time required to reach an acceptable solution becomes increasingly prohibitive. It is not unreasonable to consider designs of tens or hundreds of thousands of individual lattice struts, which would surpass the capabilities of these methods [8]. In response to these limitations of design, there is a need for a method of designing meso-scale lattice structures that is computationally more efficient than current techniques, but does not sacrifice the performance of the design.

1.3: Research Questions and Hypotheses

The crux of a meso-scale lattice structure design problem is the optimization of the topology and sizes of the lattice struts. Although methods exist to represent and build complex lattice configurations, designing these lattice structures surpasses current capabilities. It is desirable, therefore, to streamline the design process to lower the computational burden encountered.

- Research Question #1: Can a method for designing deterministic meso-scale lattice structures be developed that is efficient enough to allow for the design of highly complex lattice structures?

Since the most taxing computational requirements of the design problem stem from optimization, it stands to reason that reducing the need for such optimization would minimize the resource requirements of the process. Extending this reasoning, the ultimate goal would be to create a method that entirely eliminates optimization from the design process. One possible way to accomplish this is to implement a process of selection from a finite set of configurations instead of implementing optimization, which has a nearly infinite number of possible solutions.

- Hypothesis #1: By utilizing a unit-truss library approach, in which individual truss configurations are chosen from a set of previously optimized conditions, the majority of optimization can be removed from the design process and replaced with a process of selection of entire unit-cells.

This reduces the computational requirements of design since each lattice strut need not be individually considered.

If such a method is to be useful to the designer, it must have better performance than currently available methods. For lattice design, this performance is measured by the time required to arrive at a solution and the structural performance of the solution itself. The first research question and hypothesis address the time required for the design process. If the first research question is satisfied, the design method must also achieve results that perform nearly as well or better than current practice.

- Research Question #2: If a method exists for designing lattice structures with reduced need for optimization, can such a method be implemented without significantly degrading performance of the final design compared to current design methods?

Performance in lattice structures, which can be roughly defined as the stiffness to weight ratio, is governed by the lattice topology and the individual strut sizes. Currently, both of these attributes are derived in one of two ways. The first is to determine the topology, or the strut sizes and connectivity, through optimization. This process is incredibly computationally taxing, which limits its implementation to small design problems. For larger problems, the truss configuration is chosen by the designer, and all of the struts are set to the same size. This removes the limits set by the need for optimization, but results in a design of limited performance. If some knowledge of the

design requirements of individual portions of the model could be imparted to this process, it would be possible to design large and complex lattice structures that performed better than those with arbitrary topology and a single strut-size without requiring the use of optimization.

- Hypothesis #2: Solid body analysis of the geometry of a part can be used to guide the design process by matching individual unit-cells in a component with the corresponding stress conditions in the solid body. This information can then control the selection and sizing of components from the unit-cell library.

1.4: Validation of the Design Method

A measurement standard and method for validation must be defined in order to gauge the success or failure of these hypotheses. Thus, a series of questions, or tests, have been developed to gauge the ability of the research hypotheses to satisfy the research questions.

The first research hypothesis was that, by “utilizing a unit-truss library approach, in which individual truss configurations are chosen from a set of previously optimized conditions, the majority of optimization can be removed from the design process and replaced with a process of selection of entire unit-cells. This reduces the computational requirements of design since each lattice strut need not be individually considered.”

Proof of this hypothesis requires successfully answering two questions. First, can unit-cells be optimized in such a way that they perform best under certain conditions? In

other words, there must be a clear difference between unit-cells designed for different applications. Second, can such specialized unit-cells be used for practical design in such a way that each individual strut of the unit-cell need not be optimized? Thus, is it possible to scale unit-cell struts as a whole simply by scaling the unit-cell itself? These questions, or tests, are summarized below:

- Tests for hypothesis #1
 - 1) Unit-cells can be specialized to exhibit certain characteristics through optimization
 - 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)

The second research hypothesis theorized that “solid body analysis of the geometry of a part can be used to guide the design process by matching individual unit-cells in a component with the corresponding stress conditions in the solid body. This information can then control the selection and sizing of components from the unit-cell library.”

This hypothesis, too, must satisfy several tests: The first of these tests is simply, is it possible to use information from solid body analysis of the geometry of a part in order to create lattice structures? No formal mechanism exists for the transfer of such information; thus, some method must be developed for its effective implementation. The second test is, given that information from a solid body analysis can be used to create

efficient lattice structures, do those structures perform on par with structures designed using other methods? Satisfying this requirement is vital to proving the utility of the design process. The tests for hypothesis two are summarized below.

- Tests for hypothesis #2
 - 3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design
 - 4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods.

Table 1-1 identifies which research tests will be explored in each chapter. Chapter three, which details the creation of a specialized unit-cell library, will attempt to demonstrate that specialized unit-cells can be identified for various structural situations. It will also illustrate how such unit-cells can be implemented to avoid strut-level optimization. Chapter four will explore the feasibility of using information from a solid body analysis of the structure geometry to guide the creation of lattice structures. Chapter five, through the presentation of several example problems, will demonstrate the utility of the process as a whole, and show that its use does not negatively impact the performance of the resulting structures.

Table 1-1: Hypotheses Verification Outline

Chapter \ Research Test	1) Unit-cells can be specialized to exhibit certain characteristics through optimization	2) Implementing specialized Unit-cells as a library eliminates global topological optimization	3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design	4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods
Development of the Unit-Cell Library (Chapter 3)	X	X		
Correlation to solid-body analysis and design method (Chapter 4)			X	
Example Problems (Chapter 5)		X	X	X

1.5: Organization of the Thesis

Figure 1-2 provides an overview of the information presented in each chapter. Further detail is provided below.

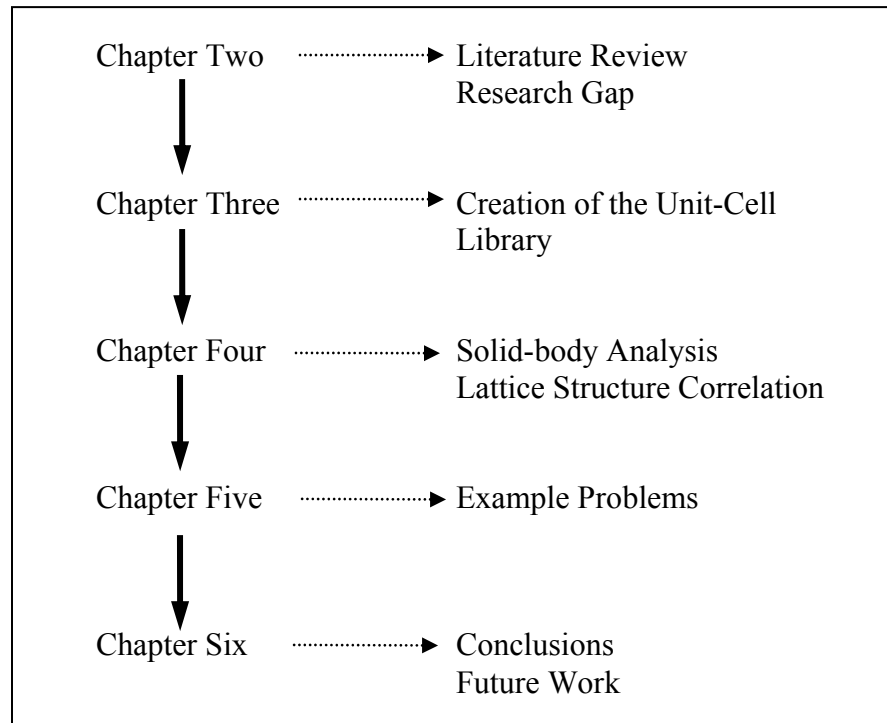


Figure 1-2: Organization of the thesis

Chapter two comprises a literature review of other work that is pertinent to this research. This includes an overview of several types of cellular structures and their application to structural design, techniques available to analyze such cellular structures, and the current approaches and methods available for their optimization. While the developments in cellular structure analysis are sufficient, advances in optimization methods have resulted in a gap between the designs that can be efficiently represented and manufactured and those that can be readily designed. The chapter closes with a

proposal to develop a design method that does not require the use of optimization, due to its current deficiencies.

Chapter three will present the development of a library of specialized unit-cells. This includes the method used to create individual entries in the library, and how such entries might be applied for design through a selection routine. This chapter concludes with a presentation of the unit-cell library in its current implementation, as well as potential avenues for extension into other realms of design.

Chapter four details the method that has been developed to correlate solid-body analysis of the design geometry to the design of the desired lattice structure. This includes the problem formulation used to select and size unit-cells into the structure from the unit-cell library, as well as several factors that must be considered and dealt with during such a process.

Chapter five presents three complete examples of the process. The first example is a very simple design problem, which allows current methods of optimization to be directly compared to the method developed in this work. The second example is a more complex example that duplicates an example problem found in literature for a new method of lattice optimization. This example is large enough that the deficiencies of current optimization methods begin to become apparent. The final example is complex enough that current optimization methods can no longer be reasonably utilized for design, which will illustrate that the utility of the method developed throughout this thesis extends beyond the capabilities of existing methods. The chapter concludes with a summary and brief analysis of the results from the three examples.

The final chapter draws conclusions based on the analysis of the experiments that were presented. The limitations of the method are identified, as well as the potential for future development.

Chapter 2: Literature Review and Gap Analysis

2.1: Literature Review

During the course of this work, several aspects of cellular structure design were researched and investigated. Deterministic lattice structures were chosen as the basis for the cellular structures in this work, although several other types of cellular structures are common. The nature of these structures is discussed, as well as the choice of deterministic lattice. To determine the performance of the structures designed throughout the course of this work, an analysis method must be chosen. To make this decision, pinned-joint analysis and beam analysis are studied, with beam analysis through the unit-truss method being identified as preferable. Finally, optimization methods and techniques are considered to provide baseline comparisons for the creation method that has been developed.

2.1.1: Cellular Structures

Cellular structures, such as closed cell foams, are used in structural applications due to their high stiffness to weight ratio. Such lightweight structures are becoming increasingly desirable in applications in aerospace and transportation [19], since weight is directly related to operating costs. The advent of rapid manufacturing has allowed for the development of a new class of cellular structures. These so-called “designed cellular structures” allow for the efficient transfer of designs from paper or computer to realizable solid, without many of the manufacturing constraints that previously governed such a

process [35], [13]. This has resulted in the advent of cellular structures whose properties can be individually tailored throughout the design domain.

Regardless of the manufacturing method, cellular structures tend to fall into one of two categories: stochastic structures, and designed structures [7]. Stochastic cellular structures include such applications as foams, whose characteristics can be controlled, but not explicitly defined. For example, when casting a foam-reinforced product it would be possible to make the voids of the foam larger or smaller, thus changing the density. It would not be possible, however, to know exactly where each void of the design will occur. The advantage of such materials is that their design and manufacture is extremely fast and relatively low-cost [15].

Designed cellular structures, on the other hand, are of a form that can be explicitly defined and analyzed [25]. These structures include honeycombs, which usually only have geometrical variations in one direction, and lattices, illustrated in Figure 2-1, which have variation in three dimensions.

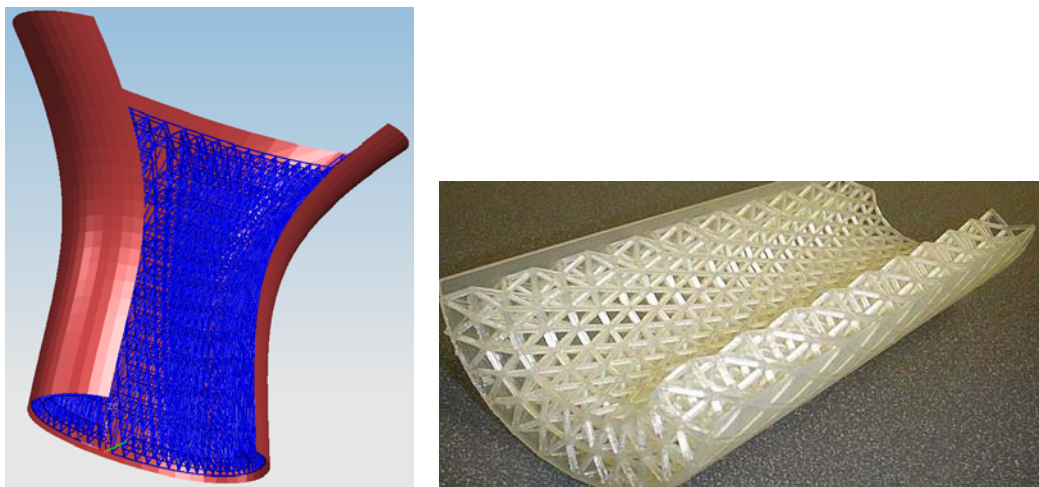


Figure 2-1: Two examples of designed lattice structures

Although they are more rigorous to design and produce, Wallach and Gibson [30] contend that deterministic lattice structures have higher stiffness and strength than

stochastic cellular structures of the same relative density. Deshpande et al [10] concur, stating that the strength of lattice structures scales as ρ , whereas the strength of foams scales as $\rho^{1.5}$, where ρ is the relative density of the structure. To illustrate this effect they note that, for a given relative density of $\rho=0.1$, a lattice structure is approximately three times stronger than a corresponding foam structure. This reflects a difference in the underlying deformation occurring in each structure. Cellular foams are dominated by the bending of cell walls, whereas lattice structures are dominated by the stretching or compression of material [9]. Because of their higher strength to relative density ratio, this work will focus on lattice structures and improvement of the design methods for their design.

2.1.2: Lattice Structure Analysis

Some form of analysis must be employed in order to accurately model and quantify the performance of cellular structures. Thus, much research has gone into determining effective methods of analysis for various cellular structures [6], [10], [19], [31-28]. Of particular note, although potentially tangential to this work, is a comprehensive review of the analytical modeling, structural mechanics, and yield characteristics of various metal honeycombs by Wang and McDowell [32].

Traditionally, analysis of truss and lattice structures has been attempted under the assumption that struts undergo only axial loading (pin-pin joints) [6], [31]. Wallach and Gibson [31] applied such an analysis to determine the structural characteristics of lattice sheets undergoing axial loading in the x, y, and z directions. Chiras *et al.* [6] extend the procedure to include analysis of similar structures undergoing bending, shear, and compression loading. Both works include experimental results to provide comparison to

the theoretical values, although Chiras *et al.* focus primarily on the mechanics of the physical samples and provide considerable information concerning the construction and quality of the experiment samples. Wallach and Gibson's work is more concerned with the theoretical analysis, which models the elastic properties of the structure with percent errors ranging from 3% to 27% (depending on the direction analyzed). Little discussion is given as to whether these models are acceptable, or where errors may arise.

Both Wallach and Gibson [31] and Chiras *et al.* [6] consider lattice structures comprised of a sheet of unit-cells that is one unit-cell thick. Deshpande *et al.* [10] consider a more general approach for an analysis of an “octet-truss” lattice, a unit-cell that is shown in Figure 2-2.

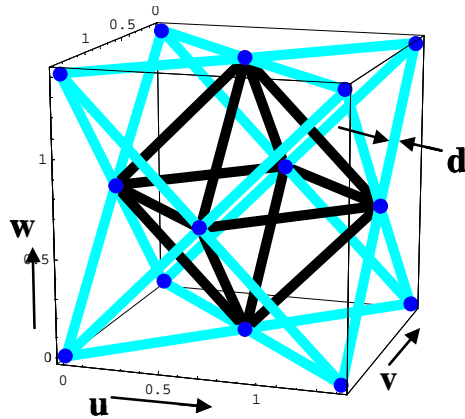


Figure 2-2: Octet-truss unit cell

This more general method, although still based on an assumption of axial strut loading, results in effective mechanical properties for individual unit-cells. By analytically combining unit-cells, any arbitrary combination of cells can be analyzed in a method similar to that utilized for finite element analysis. Johnston *et al.* [19] provide a more comprehensive analysis, considering an assumption of beam-type behavior for each lattice strut. Their unit-truss lattice model is able to simulate unit-cells under compression with a relative error of under 10%. Wang *et al.* [33] illustrate that the unit-truss analysis

method can easily be applied to practical problems of lattice design. This method does not consider the possibility of buckling in struts within the lattice structure. This does not, however, prove to be a limitation for design. A 10mm lattice strut of SL5510, a common stereolithography material, with a circular cross section of 1mm has a critical Euler buckling load of approximately 59N. For the types of structures considered during this work, the loading conditions are such that the risk of buckling is small. In deference to its increased accuracy, the FEA unit-truss analysis method developed by Johnston *et al.* will be utilized throughout this work.

2.1.3: Optimal Lattice Structures

Near the turn of the last century, Australian engineer George Michell published what would eventually become Michell's theorem [21]. This theory defines the existence of an analytically optimal truss structure under certain loading conditions. One such analytically optimal truss structure is illustrated in Figure 2-3.

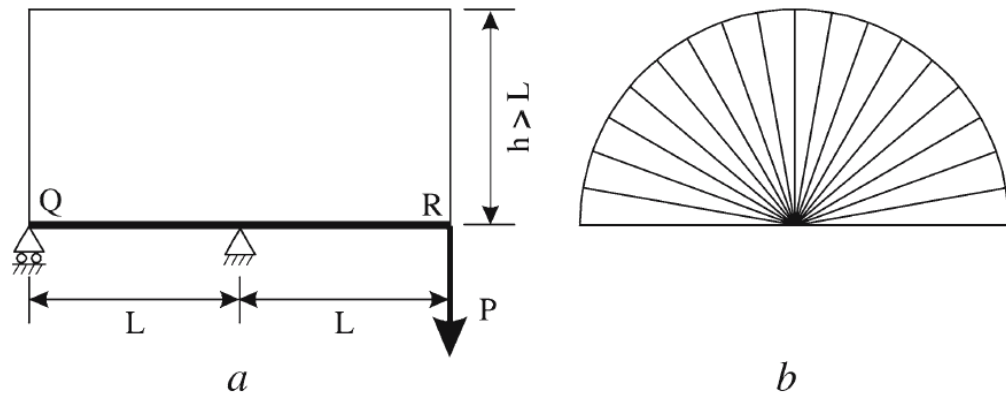


Figure 2-3: One of Michell's 1904 solutions, from [27]

Much attention has been given to Michell trusses, such as extensions to consider designs with multiple materials [11], non-linear situations [28], or designs with pre-

existing elements [27]. Unfortunately, Michell trusses often result in “a kind of framework with continuous distributions of members,” which can be considered a “truss-like continuum” [40]. Such a continuum, while analytically valid, is not conducive to practical manufacture. Although some work has been accomplished creating a discretized formulation [40], the application of these structures remains limited. Additionally, since the theory has not been extended to three dimensions, it cannot be applied to most practical applications. This lack of an analytical solution to create optimal lattice structures results in the use of optimization methods and routines for lattice structure design.

2.1.3-1: Nomenclature

Before discussing optimization techniques in detail, an issue of nomenclature must be resolved. Throughout the literature, several different phrases and names have been developed for various optimization techniques, some of which are ambiguous. Bendsøe and Kikuchi [3] refer to the development of the geometric dimensions of a continuum body as “shape optimization,” as do Allaire *et al.* [2] and Pedersen [22]. This is in deference to their use of continuums, whose optimization results often appear very organic. Zhou *et al.* [38] and Sigmund [29] refer to this process as one of “topology optimization,” thereby acknowledging the lattice-like nature of their results. Ambiguity arises from Achtziger’s [1] and Xai and Wang’s [37] use of “topology” to describe the diameters of individual lattice struts, a practice described as one of sizing by Bendsøe and Kikuchi [3] and others. Xai and Wang also utilize “shape optimization” to describe the process of moving individual nodes of a lattice structure, which Achtziger calls “geometry optimization” and Rozvany and Zhou [26] call “layout optimization.”

Achtziger [1] notes that the driving problem behind this ambiguity is that “the classical terms of geometry, topology, and sizing optimization are melting in our approach as also in many other works on this subject,” which is an apt description. An issue of sizing, for example, becomes the optimization of lattice topology if the strut sizes are allowed to approach zero.

To avoid ambiguity, the naming convention utilized by Achtziger [1] will be employed in this work: “Geometry” will refer to the layout of struts within a lattice model, i.e. the location of nodes and the struts that may connect them. “Topology” will refer to the individual diameters or cross sections of lattice struts, as well as which struts connect to which nodes. As noted by Achtziger, and in the previous paragraph, this process is actually one of combined topology and sizing, but no distinction between the two will be made.

2.1.3-2: Optimization Approach

Two basic approaches are utilized when determining geometry or topology, regardless of the particular optimization method employed. The homogenization approach, based in continuum mechanics, was pioneered by Bendsøe and Kikuchi [3]. The homogenization approach begins with a continuum of finite elements whose material densities can be individually controlled. During optimization, element densities are altered to determine an optimal shape or structure. In the final solution of the model, elements with relative densities near one are considered “present” while elements with relative densities near zero are considered “empty” or void. This method is advantageous in that it allows changes in shape (geometry) and topology without the need to remesh a

finite element model [3]. Additionally, the development of the necessary programming to create such an optimization routine can be exceedingly simple [29]. However, inherent computational errors in the finite element analysis have a tendency to create “checkerboards,” or areas of adjacent high/low density elements [38]. Ambiguity also arises in areas of the structure that do not have a clear definition of present (one) or void (zero). Various strategies have been employed to penalize the density towards a clear 0/1 material distribution [16], [38], with varying degrees of success.

The second approach, the ground structure approach, avoids the ambiguity between solids/voids by defining a lattice structure of struts and nodal connections. The initial configuration, or ground structure, represents all potential lattice topologies. Individual topologies can then be created by removing struts from the ground structure and sizing the remaining struts [12]. This approach creates structures that are easily realizable, but is limited to topology optimization alone. Achtziger [1] and Xia and Wang [37] have broadened such analyses to include geometry optimization by considering the locations of the connecting nodes as design variables. Although these methods are much faster than continuum methods such as homogenization, the final solution is still highly dependent on the choice of the initial ground configuration [1].

2.1.3-3: Optimization Method

Regardless of the approach utilized, the actual method of optimization must be established. Rozvany and Zhou [26] group methods of optimization into two categories: direct minimization techniques and indirect methods. In direct minimization techniques, such as mathematical programming, the gradients of the objective function with respect to each individual variable are calculated, and the model is updated based on these

gradients. This process is repeated until a satisfactory result is obtained. Rozvany and Zhou contend that while such methods are inherently robust, the calculation of gradients can be time-consuming, and limits the number of variables that can be effectively optimized. Indirect methods, such as optimality criteria, consider some other aspect of design to determine fitness. One such optimality criterion might be a requirement that all struts in a model be fully stressed for a given load case. The classic optimality criterion is that of Michell, whose trusses require that all struts in compression have identical stress, and all struts in tension have identical stress [40]. Such optimality criteria provide a clear relationship between each variable and its influence on the fitness of the optimization. In most situations, optimality criteria such as uniform stress are equivalent to design for minimum compliance, and provide the same solution [22].

One form of mathematical programming is least squares minimization (LSM). Least squares minimization seeks to minimize an objective function represented in the form [8]:

$$S(\mathbf{X}) = \sum_i (P_{i,target} - P_{i,actual}(\mathbf{X}))^2 \quad 2-1$$

where P_i are various aspects of the design such as volume, compliance, etc. Since the objective function is to be minimized, its derivative is set equal to zero:

$$\nabla S(\mathbf{X}) = 2 \sum_{i=1}^n \left[\frac{\partial P_{i,actual}(\mathbf{X})}{\partial \mathbf{X}} \right] [P_{i,target} - P_{i,actual}(\mathbf{X})] = \mathbf{0} \quad 2-2$$

As discussed by Rozvany and Zhou [26] the crux of such methods is calculation of the derivative of the objective function with respect to every optimization variable. The partial derivative term is the Jacobian, $J(\mathbf{X})$, of the system, and is nonlinear. Several

iterative methods have been developed to solve such nonlinear problems, including Gauss-Newton and Levenburg-Marquardt methods [23]. The latter tends to be more robust when variations in the Jacobian values are small [8]. It is for this reason that the Levenburg-Marquardt method will be applied to this research. The limitation of this approach lies in the number of variables that can be analyzed, since each additional variable adds an analysis step to each iteration [26]. Since the number of variables is tied directly to the number of struts present in a structure, increases in structure size quickly increase the number of variables that must be considered.

Particle swarm optimization (PSO) is a stochastic optimization method that could be applied to either direct minimization or optimality criteria, depending on the problem formulation. A continuation of genetic algorithms, particle swarm optimization seeks to emulate the movement of a flock of birds [20]. Each individual particle of the swarm or ‘bird’ is a configuration of the problem being studied. The movement of particles of the swarm (or the change in variable values for each member) during optimization is influenced by the history of each individual member, as well as the experiences of other members of the swarm and the swarm as a whole, as shown below [8]:

$$v_{id} = \underbrace{w_k \times v_{id}}_{\text{velocity inertia}} + \underbrace{\varphi_1 \times rand() \times (p_{id} - x_{id})}_{\text{cognition behavior}} + \underbrace{\varphi_2 \times rand() \times (p_{gd} - x_{id})}_{\text{social behavior}} \quad 2-3$$

$$x_{id}^{k+1} = x_{id}^k + v_{id} \quad 2-4$$

The velocity term of this equation is comprised of three parts: the velocity inertia, which identifies the current direction of the particle, the cognition behavior of the particle, and the cognition behavior of the entire swarm, which takes into account the best solution found by any particle p_{gd} . To identify the particle’s new location, its current

location is simply added to the new velocity term. The unit-mismatch of this operation is disregarded [20].

This optimization procedure is considered stochastic because the behavior of the swarm is governed by the pseudo-random numbers utilized to create the initial populations and velocities of the swarm particles. Thus, two identical optimization trials may achieve slightly different results. The advantage of particle swarm optimization is that it tends to be more robust when faced with optimization problems that contain many local minima [34]. Although a single particle of the swarm may be ‘trapped’ in such a minimum, other members are free to continue searching for better solutions. The disadvantage of particle swarm optimization is that it tends to be computationally expensive. While typical swarm sizes are $1/3$ the number of variables, meaning that each iteration of PSO requires $1/3$ the number of function calls as an iteration in Levenburg-Marquardt/least squares minimization, the number of iterations required to complete optimization is approximately an order of magnitude greater than that of the Levenburg-Marquardt approach[8]. This is illustrated in the example problems in this thesis.

2.2: Gap Analysis

Although much work has been accomplished in the design and optimization of lattice structures, current methods share a common shortcoming. While they are able to produce precise and accurate solutions for problems with relatively few variables, they prove too cumbersome and computationally costly when applied to problems of a larger scale. Chu *et al.* [8] demonstrate that such costs become apparent in designs with as few as 500 variables. As noted by [4] this stems from a gap between the current ability to

analyze and optimize problems. Although much work has gone into bridging this gap, it remains a substantial impediment to the practical design of lattice structures.

Rather than attempting to bridge the gap between analysis and optimization, this work proposes that it be avoided entirely. If the design of lattice structures can be accomplished through analysis only, the scale of potential designs will no longer be limited by the shortcomings of available optimization routines. Thus, the task at hand is to create a method to design lattice structures by using the information available from a non-iterative analysis process.

2.3: Summary

This chapter provided a summary of previous work relating to the design of meso-scale lattice structures. The nature of cellular solids was discussed, as were the differences between stochastic and designed cellular structures. Several different analysis methods were presented, and the unit-truss method was chosen for this work due to its ability to more accurately model structures. Michell trusses, a class of analytically optimal trusses, were shown to present an insightful solution, but one that is not practical for real-world designs. Since no analytical method exists, optimization approaches such as homogenization and the ground truss approach were investigated. The ground truss approach was determined to be preferable for these types of problems, since it extends to three dimensions easily and creates solutions that translate well to physical structures. Two methods of optimization were identified as particularly suited for this research. Levenburg-Marquardt/least squares minimization, a mathematical programming method, is particularly robust although it is limited in the number of variables it can optimize.

Particle swarm optimization is less efficient than the Levenburg-Marquardt method, but its stochastic nature allows it to be more robust to local minimums.

A research gap was identified in the methods utilized to create designed cellular structures of a large scale. While the current optimization techniques work well on a small scale, the gap between current analysis and optimization methods means such methods do not scale up effectively. Rather than trying to close the gap between analysis and optimization, which has been much studied by others, it is proposed that it be avoided by the creation of a design method that does not require the use of optimization.

Chapter 3: The Unit-Cell Library Approach

The essential problem faced with meso-scale lattice implementation is that of scale. A part that is twice as large has twice the number of lattice components, but this scales the design space exponentially. Ideally, the design problem would be greatly simplified if sections of the lattice could be decoupled from the rest of the body for a portion of the design process. Accomplishing this necessitates satisfying several requirements. Firstly, a system must be implemented that allows design and analysis of small sections of lattice within a larger structure. Such a system, called unit-cell representation, has been developed that defines the simplest lattice representation that can be repeated to create an entire lattice configuration. If a design problem is segmented into a multitude of unit-cells, various lattice configurations can be placed in those unit cells in a piecewise manner to tailor the properties of the component. Secondly, a method must exist that allows for rapid definition of the unit-cells present within a problem. I propose a unit-cell library, which consists of a finite set of unit-cell configurations from which the designer or design program can choose to populate the unit-cells within a body. Lastly, there must be a process to utilize such configurations in a way that does not require strut-level optimization. These goals directly relate to the first and second research tests, which apply to the first research hypothesis:

- 1) Unit-cells can be specialized to exhibit certain characteristics through optimization

- 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)

This chapter will satisfy these tests through a detailed description of the development and implementation of this unit-cell library approach, as well as how it might be implemented in lattice structure design.

3.1: The Unit-Cell Approach

In order to facilitate independent design of portions of the problem, it is advantageous to create a formulation that allows such small portions of the structure to be individually analyzed and manipulated. One solution to this problem is implementation of the unit-cell. The unit-cell approach allows two major operations while creating or manipulating lattice structures. First: by defining an entire structure as replicas of a single unit-cell configuration, changes can be made to the entire structure through manipulation of a single variable. This allows bulk changes or scaling to be made to the structure without necessitating redefining each individual lattice strut. Second: by defining a problem as a set of potential unit-cells, rather than any particular lattice configuration, each unit-cell can be manipulated independently of the surrounding cells. It is this second feature that proves invaluable to the lattice structure design problem.

The unit-cell definition of a model is essentially segmentation of the model geometry into areas that will later be populated with individual unit-cells. No lattice geometry is generated during the process, but rather the geometric bounds are set that

will contain the lattice elements, as seen in Figure 3-1. The motivation for this procedure is that it defines individual locations within the part that can be referenced and used to match individual unit-cells to the desired properties of the component at that location. Thus, for any given unit-cell, the design of the cell depends only on the characteristics of that location and is independent of the design of the rest of the body. The process for deriving the required characteristics of each unit-cell is explained in detail in section 4.

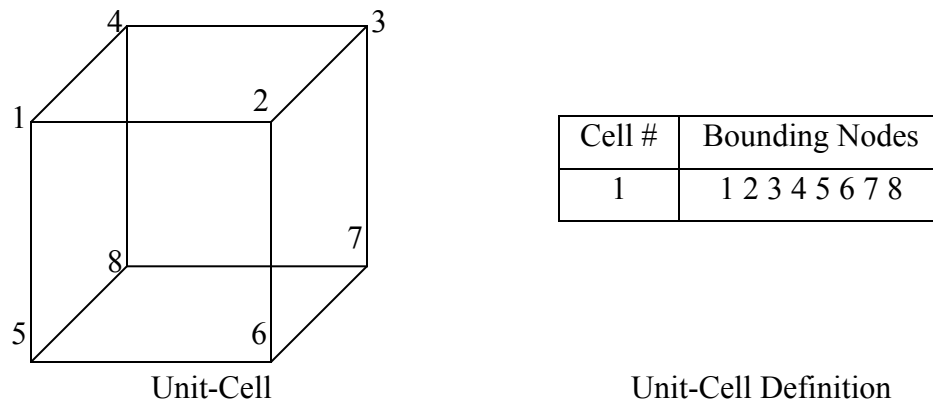


Figure 3-1: Unit-Cell Definition of a Model

The size and shape of the unit cells directly affect the final performance of the design. For this work, cubic unit cells are used that are typically sized by the thickness of the desired truss structure. Ideally, the unit-cells that compose a body would be identically sized and shaped. In practice, it is often difficult to place a predefined unit-cell into an arbitrarily shaped design. This problem is similar to that encountered when attempting to create a mapped mesh during finite element analyses; difficulties are encountered unless the design is geometrically simple.

Unit-cell definitions comprise the identification of the bounding nodes of each unit cell, as well as the nodal locations. For simple problems, the nodal definition of the unit-cells can be accomplished by creating a mapped-mesh in a finite element preprocessor, or

by straightforward analytical definition. Designs that exceed the capabilities of these methods can be defined using freemeshing or other specialized meshing approaches [14]. Regardless of the method utilized to arrive at the unit-cell definition the end result is a list of unit cells, their nodes, and nodal locations that conform to the desired model geometry.

3.2: Current Optimization Approach

The unit-truss approach allows for simple definition of a lattice structure, but has limited ability to tailor the details of the design. While unit-cells can be assigned individual configurations and strut sizes, the process must be accomplished in a piece-wise manner by the designer. This becomes tedious for more complicated designs. Optimization can be employed to avoid manual design of individual sections of truss. Such optimization, whether it consists of simply sizing the truss struts or defining the topological configuration, has previously been accomplished by analyzing the entire lattice structure as a whole. Although this technique proves useful for some design applications, the inherent complexity of lattice structures limits the scope of its application.

In general, such a ‘brute force’ approach involves creation of a unit-cell definition of the model, replication of a single configuration throughout the structure, definition of load cases, and some sort of optimization routine on the structure as a whole to determine preferred strut sizes or topology. The advantage of this approach is that it accounts for changes in loading in one unit-cell that result from design changes in adjacent unit-cells. It is for this characteristic that the brute-force approach is often employed for the design of compliant truss structures, whose large deformations and stress distributions within the

lattice structure result in a high degree of unit-cell interactions. The disadvantage of this process is that the complexity and scale of the design problem result in a plethora of local minima that pose a significant challenge for optimization.

The difficulty encountered when attempting to optimize lattice structures is that the design problem scales exponentially with the number of unit-cells in the problem. If, for example, there are N cells each with K lattice struts, the design problem has NK variables and 2^{NK} possible solutions. (In this example, each strut is assumed to have a diameter of either one or zero. This is simplistic, as when the struts are allowed to have a continuous distribution of diameters the problem of complexity is exacerbated). Therefore, each lattice strut added to the problem doubles the potential design space. Most of the potential configurations of the structure are impractical, which leads to the presence of many local minima and maxima in the design space. Identifying these local inflections prior to optimization is difficult, and they often cause improper solutions during optimization.

Several optimization methods, namely genetic algorithms and particle swarm optimization, have been developed specifically to address optimization problems with many variables and a high degree of complexity. While the specifics of each method differ, the general principle is to overcome local minima by simultaneously maintaining a relatively large set of potential solutions. The potential solutions can then be gauged against each other to judge fitness, and exhibit some degree of cross-talk to further the optimization process. The number of design sets is typically of similar order of magnitude as the number of variables in the design problem, which makes these techniques computationally taxing and slow or difficult to converge.

3.3: Unit-Cell Library Concept

Of the two dominant types of optimization undertaken during lattice structure design, geometry and topological optimization, geometry optimization is significantly more computationally intensive. Rigorously optimizing a lattice structure's geometry requires moving individual nodal locations, as well as allowing struts to be placed or removed. Although methods, such as the ground truss approach, have been developed to approximate this process in a simpler manner, they remain too complex to be practical for large lattice problems.

To further simplify the design problem, this process of optimization has been replaced by one of selection. To accomplish this, a set of unit-cell configurations and relative strut diameters has been developed that are optimized for various loading conditions. A unit-cell configuration entails particular strut topology and relative sizes. These configurations are illustrated in Figure 3-2. Development of this 'unit-cell library' enables several potential operations. If an entire structure is to be optimized simultaneously it can now be done on a unit-cell scale, with an optimization routine utilized to determine which selections from the unit-cell library are most appropriate and how they should be scaled. An even faster, if potentially less exact, approach is to utilize knowledge of the structure to select unit-cell types from the library. Thus, a tensile configuration could be selected if a section of the design is known to be under tension.

With a small amount of manipulation, in order to avoid potentially overlapping struts, it is also possible to define multiple unit-cell configurations within a single unit-cell. Continuing the previous example, such implementation might consist of a cell undergoing both tension and shear. This ability to consider multiple configurations for a

single unit-cell is pivotal to the correlations drawn between solid bodies and lattice structures in chapter four.

3.4: Specialized Unit-Cell Creation

Before a unit-cell library approach can be implemented, it is first necessary to define the individual unit-cell configurations. This process entails both topological and shape optimization. Various optimization approaches can be utilized, but the end result of any approach is to create unit-cell configurations for various loading types that may be present in the final lattice structure. The approach presented below utilizes the ground truss approach and particle swarm optimization to derive these configurations.

The premise for the creation of specialized unit cells is to design truss structures that are customized for various loading conditions. To accomplish this, individual unit cells are modeled and optimized utilizing particle swarm optimization under controlled loading conditions. The problem formulation for this process is as follows:

Given: Specified load and boundary conditions
Find: Truss strut diameters/topology
Satisfy: Minimum and Maximum Diameter constraints
Minimize: Truss volume and deflection

For compressive loading along the x-axis, such formulation would take the following form:

Given: Nodes at X=0: fixed in all DOF, all other nodes: -10N applied in X
Find: D_i , the diameters of the i struts present in the model
Satisfy: D_{\max} , D_{\min}
Minimize:

$$\min_D f(x) = w_d \times d + w_v \times V_{norm}$$

3-1

Where: D are the truss strut diameters, w_d and w_v are weighting values, d is the sum of nodal displacements and V_{norm} is the volume, normalized to an initial configuration.
or simply:

$$\min_{D \in [0.001, 5]} f(x) = (w_d \times \sum \sqrt{(dx_j^2 + dy_j^2 + dz_j^2)}) + (w_v \times V / 500) \quad 3-2$$

Where: w_d and w_v are equal to one and the values for dx , dy , dz , and V , given values for D_i , are determined through finite-element analysis of the model. Note that, for this example, the volume was normalized based on an initial volume of 500, which was the volume when all strut diameters were set equal to one.

The minimum value for strut diameters is constrained at 0.001, instead of zero, in order to maintain mathematical stability of the finite-element code utilized to analyze the model during optimization. After the optimization is complete, struts whose diameters fall below a lower threshold $D_t=0.1$ are removed from the structure, yielding a reduced number of individually sized struts. To abstract the strut diameters beyond the load magnitudes utilized for optimization, the strut diameters are normalized such that the largest strut has a value of 1:

$$D_{i,\text{norm}} = D_i / D_{i,\text{max}} \quad 3-3$$

By replicating this process, unit cells can be created that are tailored to any desired load case. The goal, however, is to create specialized unit cells that can be associated with some aspect of the results of an analysis of a solid body. Creation of specialized unit cells for compressive loading in the x , y , and z directions, as well as shear loading in the xy , xz , and yz directions, as shown in Figure 3-2, creates a unit cell library that can be correlated to σ_{xx} , σ_{yy} , σ_{zz} , σ_{xy} , σ_{xz} , and σ_{yz} stress values obtained through solid model finite-element analysis.

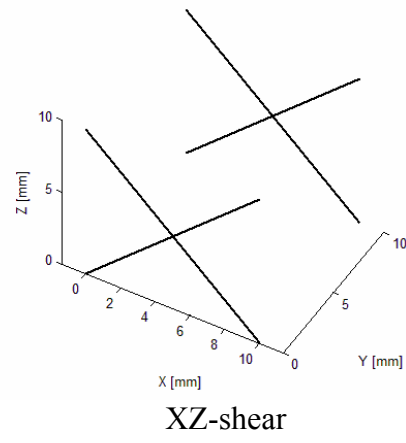
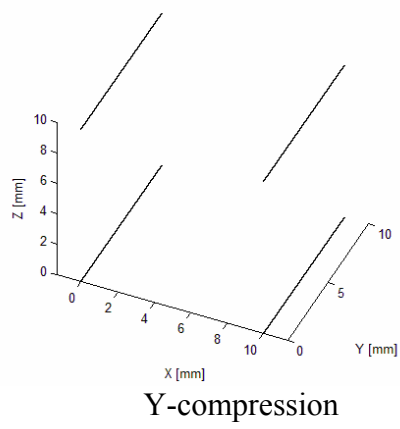
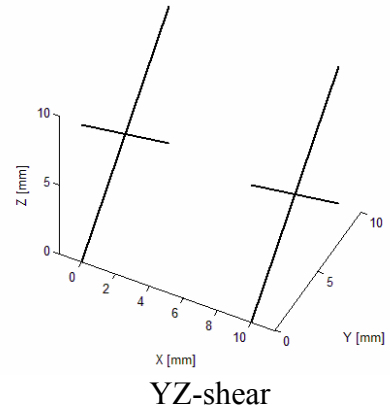
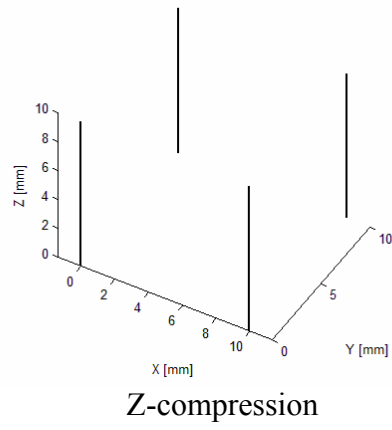
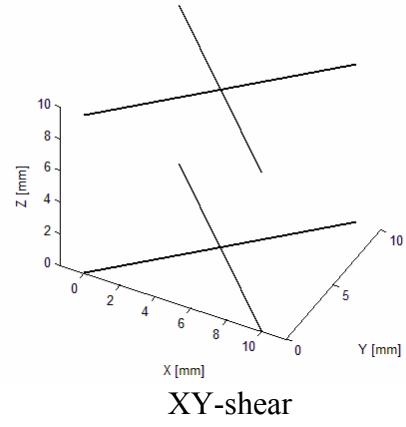
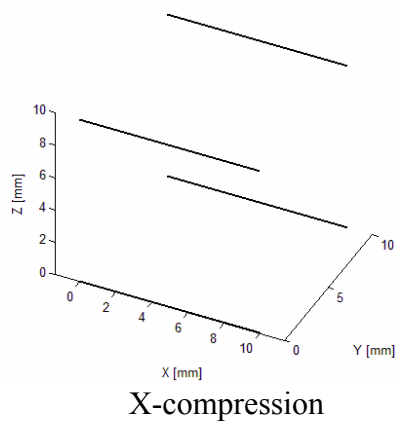


Figure 3-2: Unit cells optimized for six loading conditions

3.5: The Unit-Cell Library

Table 3-1 displays the current entries in the unit-cell library. The first six entries constitute the unit-cells that have been specialized to correlate to the stress information available from a solid-body finite element analysis. These are used extensively in the design method developed in Chapter 4. The next entry is the octet unit-cell that, as discussed in Chapter 2, has been well studied and analyzed in a closed form as well as by finite-element analysis. The Cantley truss [5], which has not been discussed extensively in this thesis, is also utilized for general loading conditions. Its open topology is efficient for manufacturing using SLS methods, since it enables the removal of excess loose powder. The last six entries exist only conceptually. Once defined, they could be correlated to higher-order finite elements, potentially resulting in improved design performance. This potentiality is discussed under future work, in section 6.3.2.

Table 3-1: The Unit-Cell Library

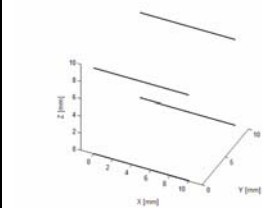
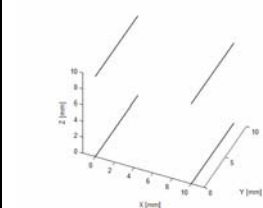
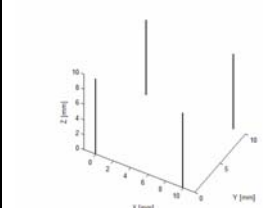
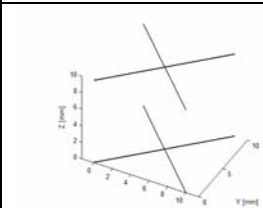
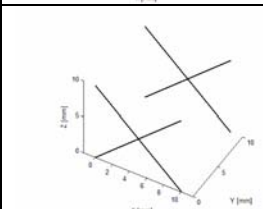
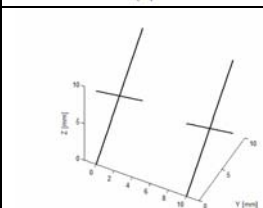
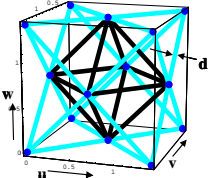

Library Category	Specialization	Notes	Image
Solid-Body Analysis Correlation	Axial Loading in X	Correlates to σ_x	
	Axial Loading in Y	Correlates to σ_y	
	Axial Loading in Z	Correlates to σ_z	
	Shear in XY	Correlates to σ_{xy}	
	Shear in XZ	Correlates to σ_{xz}	
	Shear in YZ	Correlates to σ_{yz}	

Table 3-1 (continued)

Existing Unit-Cell Configurations	Generalized loading, known closed-form analysis	Octet Truss	
	Easily built using SLS	Cantley Truss	
Proposed extended solid-body analysis correlation	Bending in X	Potentially useful for solid-body analysis correlation	
	Bending in Y		
	Bending in Z		
	Torsion around X		
	Torsion around Y		
	Torsion around Z		

3.6: Summary

This chapter enumerated the method developed for the creation of a unit-cell library. It presented the basis of the unit-cell approach, and how such an approach is beneficial to the design of lattice structures. While current design methods take advantage of the unit-cell approach during lattice definition, they fail to do so during optimization. It was stipulated that if a library of specialized unit-cell configurations could be developed, they might be implemented through selection in an optimization procedure. A method for developing such specialized unit-cell configurations through rigorous optimization was presented. Several unit-cells were identified that perform best under compression/tension and shear stress conditions. Finally, the current unit-cell library was presented and briefly discussed.

The goal of this chapter was to satisfy the first and second research tests, which applied to the first hypothesis:

- 1) Unit-cells can be specialized to exhibit certain characteristics through optimization
- 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)

It was shown that unit-cells can be specialized to perform best under certain conditions. The current unit-cell library contains configurations specialized for several types of stress, but configurations could also be developed based on thermal characteristics, resonance response, or other design domains. A general method was also

conceptualized for utilizing unit-cell in such a way that individual struts need not be optimized. By considering entire unit-cells as the optimization target variable, several lattice struts can be topologically optimized and sized as a single variable. By allowing selection of different unit-cells from the library, instead of simply considering unit-cell scales, a certain amount of lattice geometry optimization could also be undertaken.

This chapter has satisfied the first research test, and lent credence to successful achievement of the second research test, which will be further investigated in chapter five.

Chapter 4: The Design Process

Although the unit-cell library approach greatly simplifies the process of topological design, several hurdles must still be overcome to realize a simple design method for lattice structures. While the unit-cell library approach allows selection from a pre-defined set of unit-cells, some sort of decision must be made concerning the selection process and sizing of the individual unit-cells in a body. Optimization can be conducted to do so but, while it is less complex than a strut-oriented approach, remains a limiting factor to design. Manual selection and sizing of members from the unit-cell library is feasible, but relies on the designer to select appropriate configurations and sizes and is thus prone to degradation of the resulting design performance.

It is desirable to develop a method to guide the selection and sizing of entries from the unit-cell library into a lattice structure design without an intermediate optimization process. This chapter enumerates such a method for automatically selecting and scaling entries from the unit-cell library, and placing them within a lattice structure design, based on finite-element analysis of the bounding geometry and loading conditions of the problem. Care has been taken during development of the process to eliminate the need for optimization during design, in an effort to reduce the computational requirements and overall design time. Development of this design process will satisfy the third research test, which is for hypothesis #2: can solid body analysis be utilized to select and size unit-cells from the library during lattice structure design?

4.1: Motivation and Concept

Since lattice structures are typically implemented in areas of design where weight concerns are paramount, it is often crucial that the structure be as efficient as possible. While there are multiple approaches that might be utilized to accomplish this, the general problem formulation for design is as follows:

Given: A specified ground structure and loading conditions

Find: The lattice topology/sizes

Satisfy: Maximum/minimum strut diameter constraints, volume constraints, deformation constraints

Minimize: Volume and/or deformation

A mathematical formulation of this problem is:

Given: i lattice struts, with f_k applied loads/boundary conditions

Find: D_i , the strut diameters

Satisfy: D_{\max} , D_{\min} , V_{\max} , d_{\max}

Minimize: $\min_D f(x) = w_d \times d + w_v \times V$

Where D are the truss strut diameters,
 w_d and w_v are weighting values,

$$d = \sum (dx_k + dy_k + dz_k) : \text{the sum of displacements} \quad 4-1$$

$$V = \sum r_i^2 l_i \pi : \text{the volume.} \quad 4-2$$

Note that this formulation concerns topology if $D_{\min} = 0$, or is simply an issue of sizing if $D_{\min} > 0$, and that the parameter being minimized often includes the displacement, as above, as well as compliance, or strain energy.

Previously, this problem has been solved through rigorous optimization of the lattice structure. This process considers the diameter of each strut as a variable in an optimization procedure. This is often computationally impractical, but can be avoided if

the design of individual portions of the lattice is decoupled from the rest of the structure. Doing so requires knowledge of two aspects of that particular part of the structure: the types of stress present and the relative magnitude of those stresses in relation to the rest of the body. This information simplifies the design process, since it is possible to design any arbitrary portion of the structure without regard to the rest of the model. Such a procedure can be implemented on the unit-cell level, essentially by utilizing the same approach used to develop entries in the unit-cell library. That is, each individual unit-cell of the model could be optimized based on the problem formulation above and the expected stresses of the design. This decoupling process transforms the design problem as follows: instead of solving a single design problem composed of N unit-cells each with K struts and a total of 2^{NK} potential solutions, the design problem now consists of N independent problems each with 2^K possible solutions. (Note that this simplified illustration allows only strut sizes of zero or one, a continuous distribution further adds to the complexity).

While such decoupling of unit-cells reduces the complexity of the design, it still requires optimization of the individual cells. Implementation of the unit-cell library can avoid such optimization if individual entries within the library are correlated to specific stress information available from finite-element analysis. If, for instance, previous analysis yields knowledge of three axial stresses and three shear stresses for each unit-cell, unit-cell configurations can be selected from the library that are already optimized for such loading conditions. These configurations can then be appropriately scaled based on the stress magnitudes and relationships to other unit-cells in the body. This second step further reduces the simplified example design problem to NK independent problems,

each with 2 possible solutions. The impact of utilizing different design approaches on design complexity is summarized in Table 4-1.

Table 4-1: Complexity of Different Lattice Design Approaches

N= # unit-cells K = # struts/cell	Problems	Dimensions/ Problem	Possible Solutions/ Problem
“Brute Force”	1	NK	2^{NK}
Unit Cell	N	K	2^K
Unit Cell + Library	NK	1	2

The crux in the development of such a design approach lies in defining a specific method for stepping from finite-element stress data to unit-cell library entries and scales. The approach that has been developed, whose details are enumerated in the following sections, entails specific steps that can be applied to any arbitrary lattice design.

Figure 4-1 illustrates the Unit-Cell Library Approach for the design of meso-scale lattice structures, and serves as a guide to the rest of the chapter. The first step in this process is to use the problem definition, section 4.2, to create a solid body analysis, section 4.3, and unit-cell model, section 4.4. The information created during analysis of the solid body must then be correlated to the unit-cell model, section 4.5. This allows the lattice topology to be defined in section 4.6. Several ambiguities resulting from the topology generation are resolved in section 4.7, which leads to a final structure definition in section 4.8. Section 4.9 will conclude the chapter by providing a summary of the advantages and consequences of the design process.

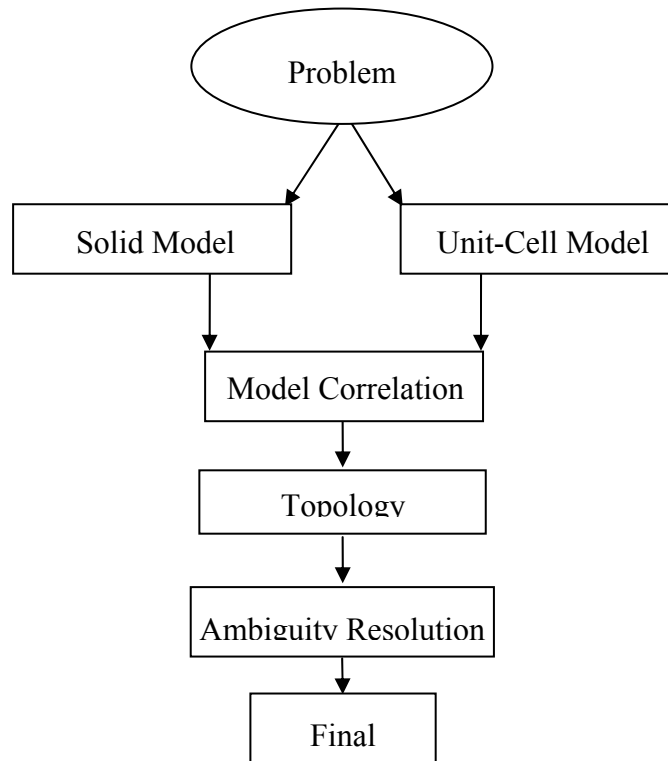


Figure 4-1: Unit-Cell Library Approach

4.2: Problem Definition

The problem definition begins the design process, and serves as a guide throughout. It consists of two parts; the main component of the problem definition is the “bounding geometry” of the problem. This is the volume, or area in the case of two-dimensional trusses, that the structure is allowed to occupy. For simple problems, such geometry can be represented through an analytical definition, although complex problems may be parametrically modeled. This serves as a limit, rather than a requirement, as it is quite likely that the final lattice will only fill a portion of the bounding geometry.

The second part of the problem definition contains the expected loads and boundary conditions that will be applied to the structure. Since they will motivate the

specialization of the lattice structure, careful consideration must be given to ensure that the loads and boundary conditions accurately represent the expected loads of the design. A graphical representation of a problem definition is shown in Figure 4-2. In this case, the model was created through an analytic definition of lines and Bezier curves. The example problem in section 5.3 contains a geometry defined through parametric modeling.

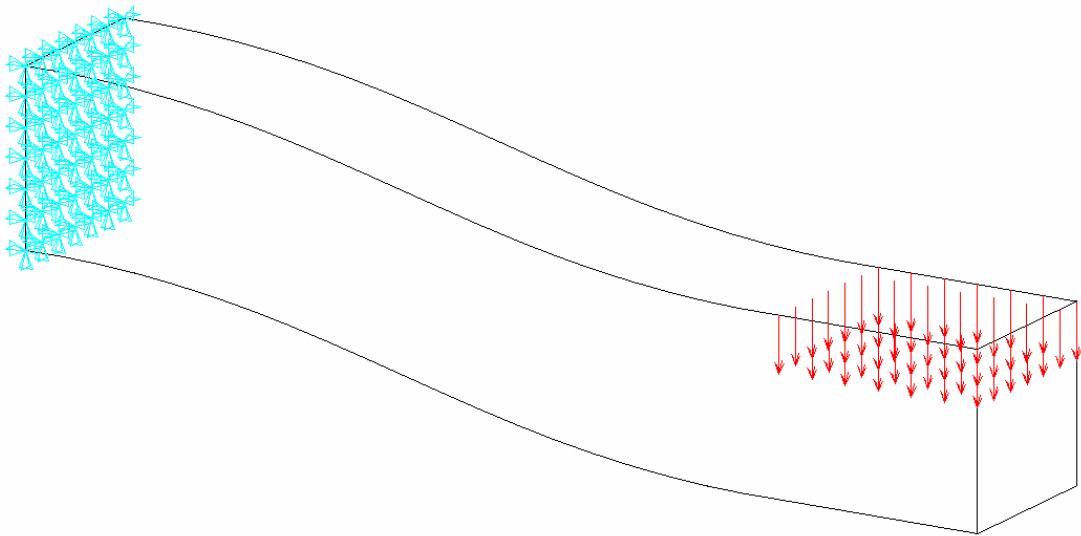


Figure 4-2: The Problem Definition

4.3: Solid Body Analysis

The second step in the design process is the creation of a solid model analysis, or analogously: solid body analysis. This is essentially a finite-element analysis of the bounding geometry created in the previous section, with application of the loading and boundary conditions previously specified. While the requirements of this design are not stringent, there are several factors that should be considered. Although little consideration

has as-yet been given to the desired lattice structure, the mesh of the solid model should be such that several of the finite-element nodes are within the area created by each lattice unit-cell. Since the unit-cells have not yet been defined, this requires only a consideration of the general scale of the unit-cells, rather than a cell-by-cell review of unit-cells and nodes. When choosing finite-element types, it is more important to select elements that will accurately model the mechanics of the structure, rather than those that are similar to the shapes of the unit-cells. The only factor of importance to the design process is the placement of the nodes themselves, so the types of elements used for analysis are arbitrary. An example solid body analysis showing finite-elements, applied loads, and boundary conditions is illustrated in Figure 4-3.

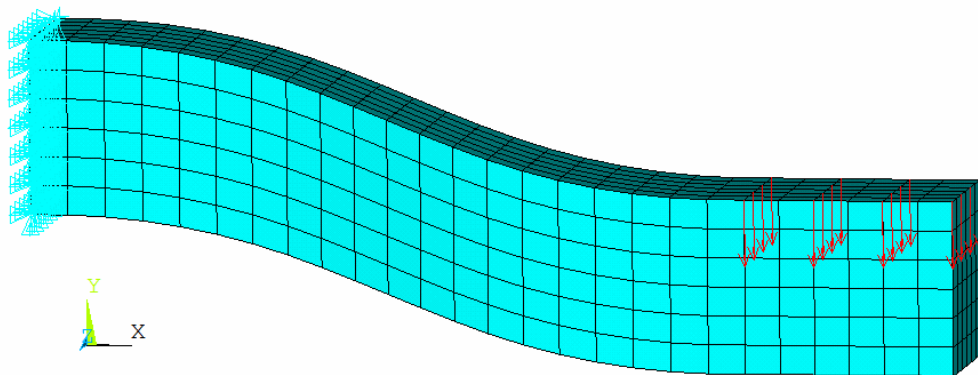


Figure 4-3: Solid body analysis

The result of the solid body analysis is the stress distribution throughout the body, shown in Figure 4-4. This information is exported as a list of nodal locations and nodal

stress values, which will be utilized in the model correlation process described in section 4.5.

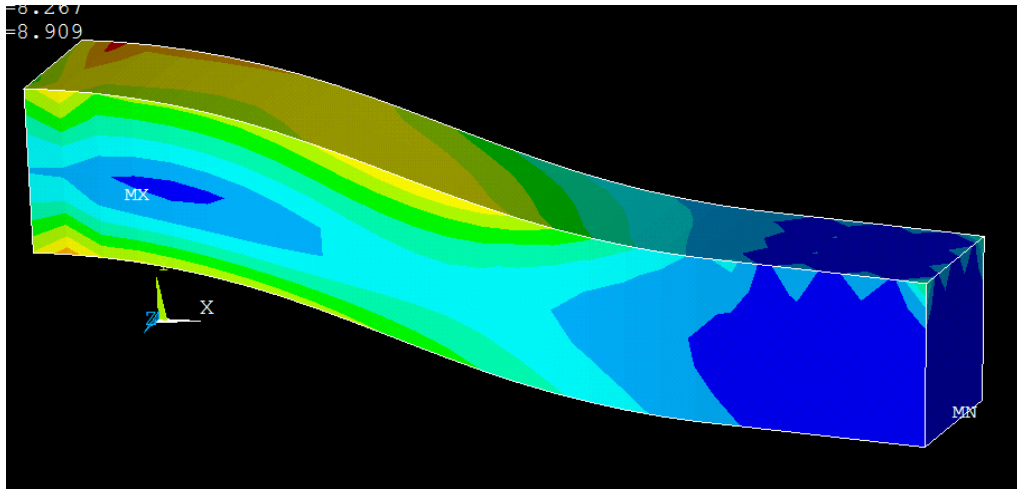


Figure 4-4: Stress information from finite-element analysis

4.4: Unit-Cell Model

The purpose of the unit-cell model is to explicitly define the locations of the unit-cells that will be used to create the lattice structure. While consideration was given to the general sizes of the unit-cells during the solid body analysis, this step defines individual unit-cells. This is purely a computational preparation, as no lattice struts are created or defined at this time. An idealized representation of this step is illustrated in Figure 4-5, although this process generally does not produce graphical results.

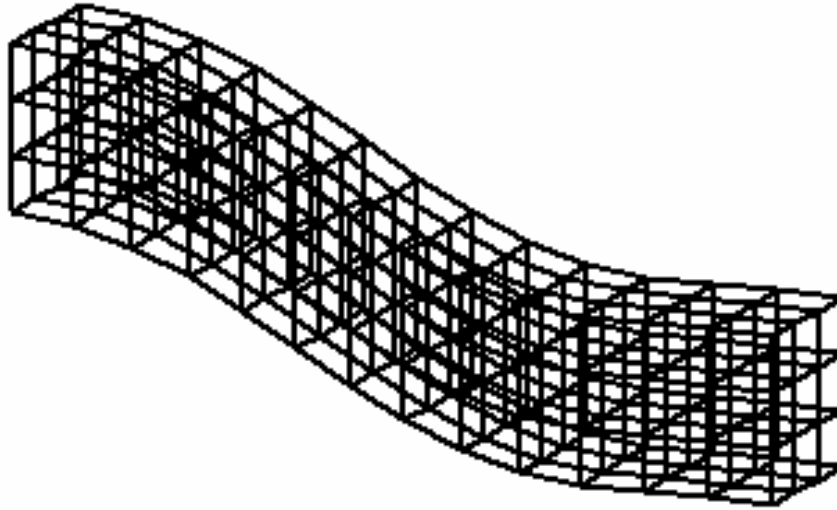


Figure 4-5: Representation of model defined as a set of empty unit-cells

Unit-cells are created by subdividing the model into cells of the desired size. Essentially, the process is identical to the creation of a mapped mesh for finite-element analysis, with similar issues arising during the formulation of complex problems. In cases where the problem definition has been analytically defined, this process can often be completed analytically. For example, since the original model shown in Figure 4-5 was defined by a series of three-dimensional Bezier curves, a unit-cell model was created by subdividing the volume with a series of identical curves with varying offsets, segmented to the desired unit-cell sizes. For more complex problems, a method has been developed to create a unit-cell model definition by offsetting an arbitrarily complex surface [14].

Rather than a graphical representation, this data is stored as a set of the nodes that make up each unit-cell, similar to a finite-element definition. Simply put, each unit-cell is defined by its bounding nodes, and each node has given coordinates. Table 4-2 provides an abbreviated unit-cell model definition.

Unit-Cell	Nodes							
1	1	2	3	4	5	6	7	8
2	2	9	10	3	6	11	12	7
3	9	13	14	10	11	15	16	12
4	13	17	18	14	15	19	20	16
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:

Node	X	Y	Z
1	0	0	0
2	1.3	0	0
3	1.3	1.4	0
4	0	1.4	0
5	0	0	1.2
:	:	:	:

Table 4-2: Unit-Cell Model Definition

4.5: Model Correlation

Further progression of the design requires that the stress information exported from the solid body analysis be correlated to the unit-cell model. This will provide stress information associated with each unit-cell that can be utilized for topology generation. The first step in this process is simply to identify which nodes correlate to which unit-cells. This is accomplished by searching the nodes of the solid body for nodes whose location falls within each unit cell. This is shown below in equation 4-3, where N are the nodes, and V_i are the extents of the i unit-cells. The stress values are then averaged, to provide average stress values for each unit-cell.

$$N^i = N \in V_i \quad 4-3$$

The result are six average stress values for each unit cell: σ_{xx} , σ_{yy} , σ_{zz} , σ_{xy} , σ_{xz} , σ_{yz} . Since the average stress values are based on a solid model of the bounding geometry, the values themselves provide little information. What is more useful is the distribution of the stress values throughout the model. To generalize the stress values, and remove the

specificity of the solid body analysis, they are normalized to provide scaling values such that the largest value present in the model is equal to one.

$$S_{i,j}^u = \text{avg}(\sigma_j^i) / \text{avg}(\sigma_j^i)_{\max} \quad 4-4$$

In the above equation, $S_{i,j}^u$ are the six, j , scaling factors for each of the i unit cells, σ_j^i , are the stress values associated with the i^{th} unit cell for each of the j stress directions, and $\text{avg}(\sigma_j^i)_{\max}$ is the maximum average stress present in the model in *any* direction, not just the direction of the σ_j^i stress values. This means, for example, that a model undergoing primarily axial loading would have axial scaling factors near one, and shear scaling factors much less than one.

The result is a distribution of six scaling values that reflect the magnitudes and types of stress at each unit-cell throughout the model. These six scaling values correlate to the six entries in the unit-cell library, shown in Table 3-1, that were specialized for such types of stress. This correlation between the scaling values and the entries in the unit-cell library will be utilized to scale the lattice struts during the topology generation phase of the design process.

4.6: Topology Generation

Once the six scaling factors have been determined for every unit-cell in the model, it is possible to define the struts and diameters for the lattice. Before this can be accomplished, however, there are several factors that must be considered. The first of these is that, since entries in the unit-cell library might contain struts of different relative scales, a second scaling value must be introduced for each unit cell. With this in mind,

each strut is scaled by $S_{i,j}^u$, the scaling value presented in the previous section, and then by $S_{j,k}^L$ a scaling value specific to each strut in each unit-cell in the library. $S_{j,k}^L$ is set such that the maximum scale for any strut in a unit-cell library entry is one.

Since all of the scaling values are normalized, some method must be provided for controlling the overall size range of the lattice struts. This leads to the introduction of the maximum and minimum diameter constraints D_{\max} and D_{\min} . The maximum and minimum diameters are implemented such that a lattice strut having a unit-cell scaling value of one and a library scaling value of one would have a diameter equal to D_{\max} , while a strut having values of zero and zero would have a diameter of D_{\min} . Thus, the sizing equation for lattice struts is as follows:

$$D_{i,k} = [S_{i,j}^u \times S_{j,k}^L \times (D_{\max} - D_{\min})] + D_{\min} \quad 4-5$$

for a structure with i unit cells, j stress directions correlated to entries in the unit-cell library, and k lattice struts per unit-cell. Note that j is an index in $S_{i,j}^u$, the scaling factor resulting from the solid body analysis, as well as $S_{j,k}^L$, the scaling factors from the entries in the unit-cell library. This is because the entries from the library that will be used for design are specifically correlated to the six types of stress from the solid body analysis. A structure whose lattice struts have been sized with this method is shown in Figure 4-6; struts visibly thickened in different directions reflect the different types and magnitudes of stress present in the solid body analysis.

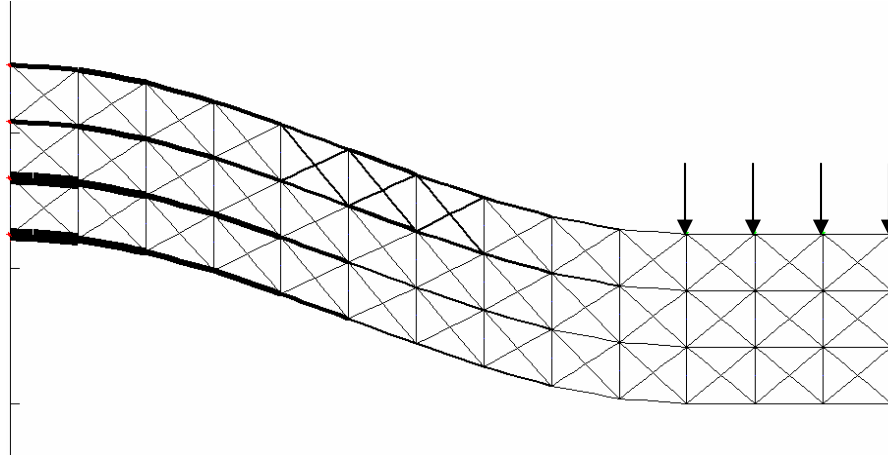


Figure 4-6: Resulting model showing sized lattice struts

While this satisfactorily sizes the lattice struts, because $D_{\min} > 0$ the lattice topology is not yet altered. To accomplish topology alterations, while retaining the ability to select a nonzero minimum diameter, a third diameter parameter is introduced. The cutoff diameter, D_c , is a diameter that falls between D_{\min} and D_{\max} . Any struts whose diameters are below the cutoff diameter are discarded, resulting in a topologically reduced structure. Equation 4-6 illustrates this, using D_i as the diameters of the struts present in the model, and *null* as an indicator that struts are to be deleted.

$$\text{if}(D_i < D_c) \rightarrow D_i = \text{null}$$

4-6

4.6.1: Setting Minimum, Maximum, and Cutoff Diameters

While D_{\min} , D_{\max} , and D_c efficiently determine the lattice topology/sizing, appropriate settings for these values are not intuitive. Since the stated purpose is to reduce the complexity of the design process, it is desirable to simplify the selection of these parameters. Ideally, a relationship between the variables would be determined that would allow two of the variables to be calculated given a set value for the third. To

investigate the effects of these values, a study was conducted for the first example problem in section 5.1. Since the first example problem was relatively simple in nature, it was possible to conduct an exhaustive search of the parameters. The results of this study are presented in this section.

4.6.1-1: Minimum Diameter

A variety of feasible minimum, maximum, and cutoff diameters were utilized to create various lattice structures, and the effects of the changes were compared to determine appropriate sizes and scales. Figure 4-7 illustrates the effects of holding the maximum diameter and cutoff diameters constant while varying the minimum diameter. It is clear that, for a given maximum diameter, a larger minimum diameter results in a stiffer structure. The limit to this trend is the volume constraint imposed by the design problem.

Given that it is advantageous to set the minimum diameter as high as possible, maximum/minimum diameter pairs can be created that result in any arbitrary lattice volume. Figure 4-8 illustrates three sets of such pairs, for designs having 500mm^3 , 1000 mm^3 , and 1600 mm^3 volumes. The data suggests that there exists a preferred minimum/maximum diameter for any given design, but that the preferred values change with the lattice structure volume. If a relationship between the maximum and minimum diameters, or the maximum diameter and lattice volume, could be developed it would be possible to create a preferred lattice structure by completing only several iterations. These iterations could be used to identify the preferred lattice with the correct volume.

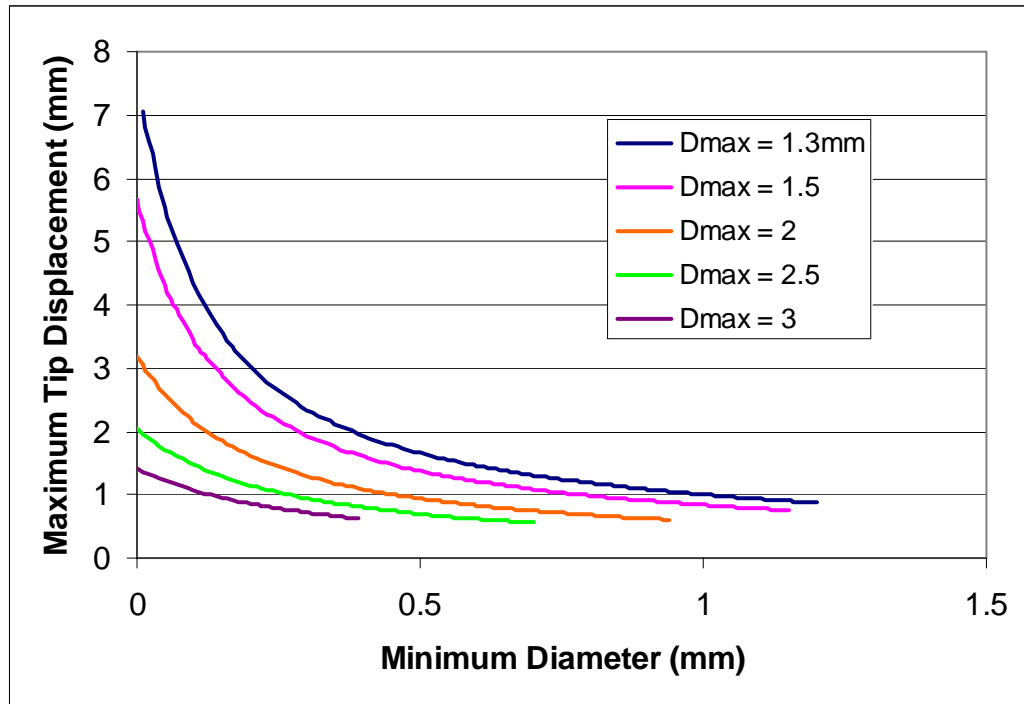


Figure 4-7: Study of the effects of minimum strut diameter on displacement

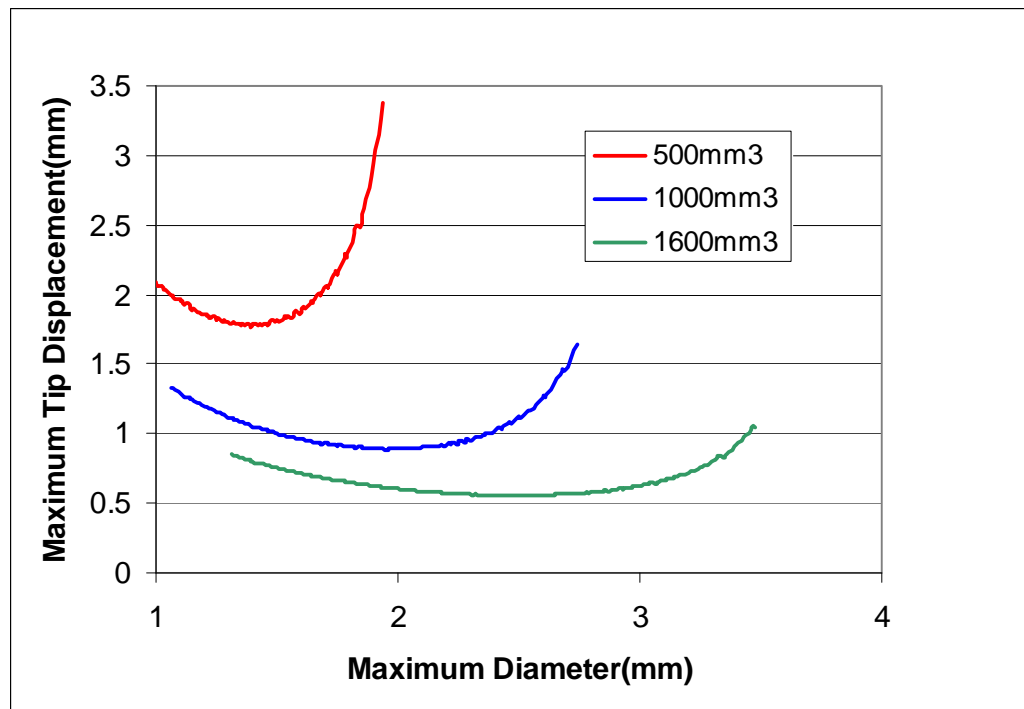


Figure 4-8: Study of the effects of maximum diameter when volume is held constant

By identifying the lattice configurations illustrated Figure 4-8 that have the least deflection it is possible to investigate the existence of a relationship that would allow creation of preferred lattice structures without optimization. Table 4-3 reveals that, for this particular example problem, preferred lattice structures have a minimum diameter that is approximately 28% that of the maximum diameter. This data suggests that lattice structure designs have a constant minimum/maximum diameter ratio that results in preferred structures, although it does not necessarily imply that all such structures have a ratio equal to 28%. Future example problems will provide more information about such a conclusion.

Table 4-3: Maximum and minimum diameters for preferred lattice structures

Volume (mm ³)	Min Diameter (mm)	Max Diameter (mm)	Min/Max
500	0.40	1.39	0.288
1000	0.55	1.97	0.279
1600	0.70	2.50	0.280

4.6.1-2: Cutoff Diameter

The other parameter in truss structure creation is cutoff diameter. This diameter is used to remove struts from the design that contribute little to the structural performance. In the current method, it is calculated as a percentage of the span between the minimum and maximum diameters. This process is represented in the equation below, where D_c is the cutoff diameter, D_{\max} and D_{\min} are the maximum and minimum diameter settings, and p is the cutoff percentage.

$$D_c = p(D_{\max} - D_{\min}) + D_{\min} \quad 4-7$$

The effect of varying the cutoff percentage is illustrated in Figure 4-9. While increasing the cutoff percentage has a slight detrimental effect on the tip displacement of the structure, the volume of the structure is more sensitive. For this example problem a five percent change in the cutoff variable resulted in a 2.6% increase in tip displacement, but a 15.0% decrease in structure volume. A 2.5% cutoff parameter seems to provide a reasonable compromise between structural stiffness and volume. This 2.5% cutoff will be utilized throughout the rest of the examples.

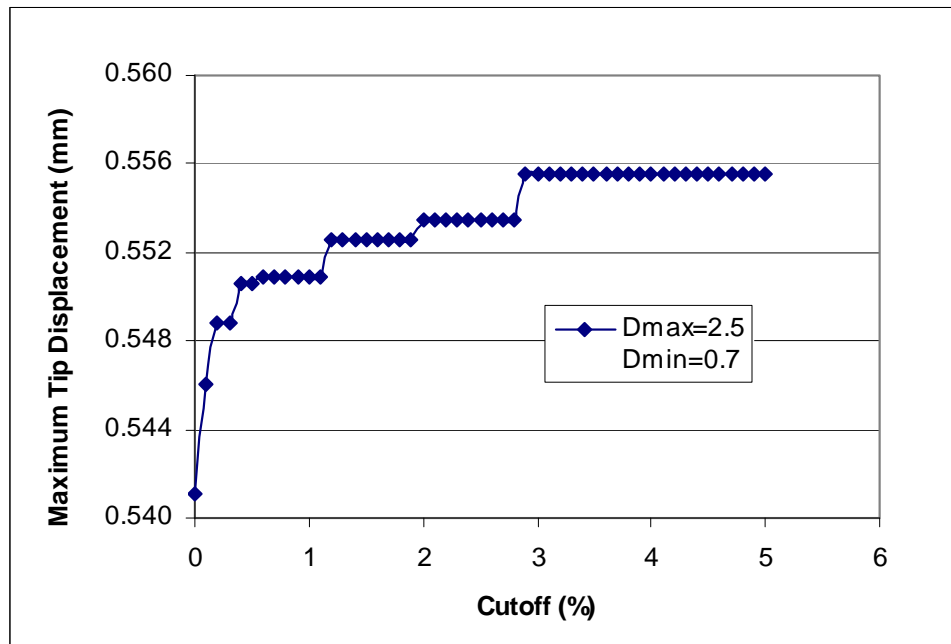


Figure 4-9: Effects of cutoff diameter on tip displacement

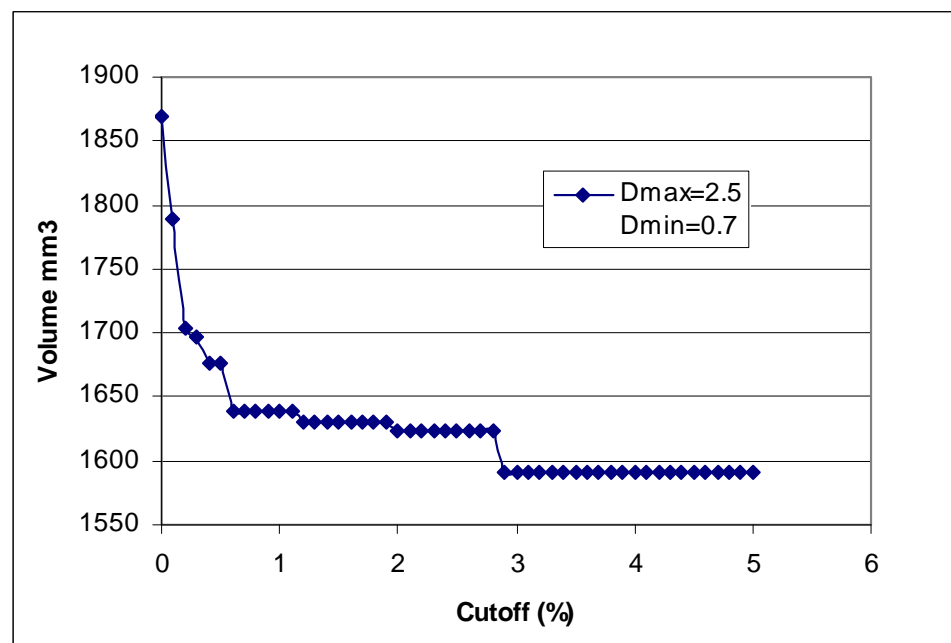


Figure 4-10: Effects of cutoff diameter on volume

4.6.1-3: Summary

The previous subsections arrived at the following conclusions: 1) Preferred lattice structures seem to have a minimum/maximum diameter ratio equal to approximately 28%. 2) A cutoff percentage of 2.5% provides a good tradeoff between decreasing the structural performance of the design and decreasing its volume. This means that, of the three variables, the only value that must be set to create a truss structure is the maximum diameter. This value can then be adjusted to minimize the volume, deformation, or other desired design parameters.

4.7: Ambiguity Resolution

Several effects of topology generation create ambiguity in the lattice structure. If two or more of the entries from the unit-cell library contain the same strut, multiple instances of the same strut can be defined within a unit-cell. Similarly, adjoining unit-cells define ambiguous struts at their shared boundaries. To resolve this ambiguity, struts are identified with identical starting and end nodes. The strut with the largest diameter is retained, and all other instances are deleted.

The process utilized to accomplish this is shown below, where $(N_{i,1}, N_{i,2})$ and $(N_{j,1}, N_{j,2})$ are the end nodes of the struts under examination, D_i and D_j are their diameters, and *null* indicates a strut that is deleted from the model.

$$\begin{aligned} & \text{if}(N_{i,1}, N_{i,2}) = (N_{j,1}, N_{j,2}) \{ \\ & \text{if}(D_i > D_j) \rightarrow D_j = \text{null} \\ & \text{else} \rightarrow D_i = \text{null} \} \end{aligned} \tag{4-8}$$

4.8: Final Structure

The final structure is stored in a similar form to other lattice design methods. This entails a list of nodes and their locations, identification of the end nodes for each strut, and the diameters of the struts. By utilizing a similar definition, structures developed through this method can easily be imported into previously developed lattice structure routines. These routines include various analysis methods as well as methods to create solid model representations such as “.stl” files, which allow for production through rapid manufacturing methods.

4.9: Summary

Although the unit-cell library allows for selection of unit-cell configurations from a finite set, development of a method of selection that does not require optimization would be advantageous. However, the performance of the resulting designs must not be unduly sacrificed in difference to a less computationally taxing design process. To accomplish these goals, a design process has been developed that selects and scales entries from the unit-cell library that have been specifically correlated to particular types of stress. Stress information throughout the model is generated from finite-element analysis of the desired bounding geometry and expected loading conditions. The entries from the unit-cell library are then scaled with regard to the stress present within any given part of the design. Development of this process has shown that it is indeed possible to create a lattice structure design method that utilizes solid body analysis to select and

size unit-cells from the library during lattice structure design, satisfying the first test for hypothesis #2.

By using stress information from the finite-element analysis to guide topology generation, the lattice structure is tailored for the needs of the problem. This mitigates the loss of performance that results from the elimination of optimization during design. The examples to follow in chapter five will demonstrate that structures designed with this approach do not undergo a significant degradation in performance compared to optimized structures, and realize significant gains in performance compared to models comprised of identically sized struts.

Chapter 5: Example Problems

The previous chapter presented the method that has been developed to design lattice structures with information gathered from analysis of the body geometry and loading, eliminating the need for optimization. This chapter will present several examples that illustrate the feasibility of this process, as well as the advantages it offers when compared to existing methods. This will address the research test #2, for hypothesis one, and research tests #3 and #4, for hypothesis two:

- 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)
- 3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design
- 4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods.

Three example problems will be presented, in increasing order of complexity. The first example problem, the simplest, will allow direct comparison between the unit-cell library approach and the optimization methods currently available. The second example, although it is in two dimensions, is slightly more complex will compare the unit-cell library method to an optimization method developed by Xia and Wang. This example will also illustrate the initial limitations of the current optimization methods. The third,

and final, example will illustrate the utility of the unit-cell library approach for problems that are too complex for other methods of lattice structure design.

5.1: A Simple Example Problem

The first example problem is comprised of a cantilever beam 50mm long, 20mm high, and 10mm wide, constructed from a material with an elastic modulus of 1960N/mm. As shown in Figure 5-1, the beam is fixed at one end with two 10N loads on each corner of the beam tip, and is to have the minimum deflection possible while maintaining a volume of 1600mm³. The resulting design is to be a lattice structure whose topology is developed through a ground-truss approach. The base configuration of the ground-truss is based on 10 unit-cells, each populated with the six entries of the unit-cell library that correlate to the stress information available from ANSYS finite element analysis (Figure 3-2), as shown in Figure 5-2.

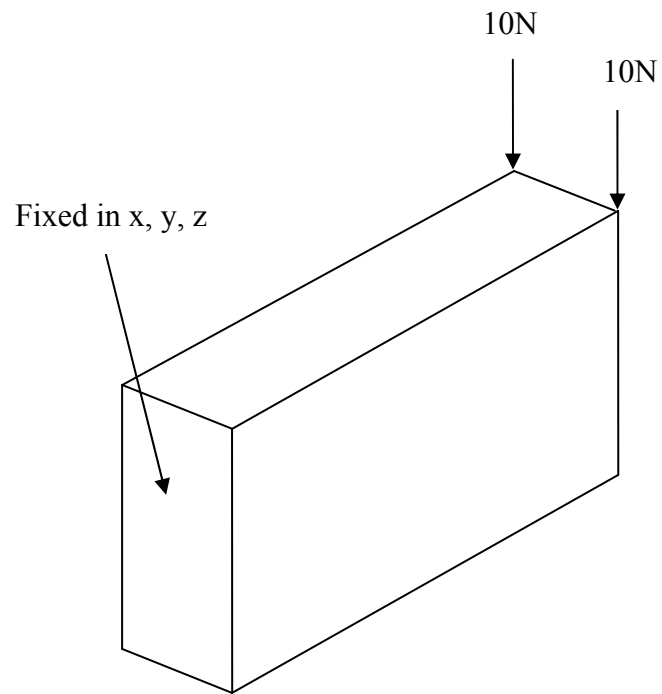


Figure 5-1: First Example Problem

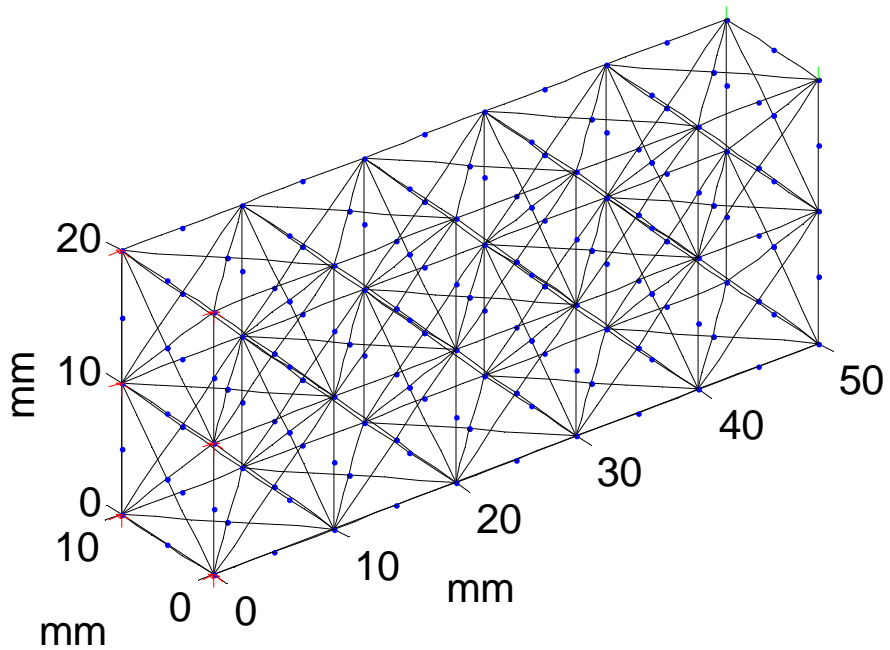


Figure 5-2: Base truss for cantilever beam example

Four methods of developing the design topology will be presented and compared. The first method utilizes all potential lattice struts, with equal diameters. The second and third methods reduce the topology by optimizing the ground-truss using particle swarm optimization and Levenburg-Marquardt/least squares minimization, respectively. Lastly, topology will be developed with the method developed in this thesis, which derives strut diameters based on solid-body finite element analysis.

5.1.1: Identically Sized Lattice Structure

Creating a lattice structure by collectively adjusting all struts is advantageous due to extremely fast creation times, but produces very compliant structures for a given volume. Performance limitations aside, this procedure can be utilized for any structure, and is often the only feasible option for large structures. In this instance, it provides a useful baseline for comparing the various lattice structure creation methods.

Defining a lattice structure with identically sized struts is a straightforward process if the base topology is previously defined. Each individual strut is simply re-defined with the diameter chosen for the design, as shown below, where i represents the various struts and D is the specified diameter.

$$diameters(i) = D \quad 5-1$$

For this example the diameter of the struts was set to 1mm, yielding a structure with a volume of 1609.6mm³, strain energy of 12.128Nmm, and a maximum tip displacement of 1.21mm. The identically sized lattice structure was defined in 0.38 seconds.

5.1.2: Particle Swarm Optimization

The second approach to deriving topology is to optimize the diameters of the individual lattice struts using particle swarm optimization. Note that such a process is shape optimization only, rather than topology optimization, as no struts are removed from the model. The problem formulation for this process is as follows:

Given: The ground truss and loading conditions pictured in Figure 5-2 where containing $i=166$ struts.

Find: D the truss strut diameters

Satisfy:

$$D_{\max} = 5, D_{\min} = 0.001 \text{ (for numeric stability)} \quad 5-2$$

Minimize:

$$\min_D f(x) = w_U \times U^2 + (V - 1600)^2 \quad 5-3$$

Where: D are the truss strut diameters,
 $w_U=100$ is a weighting value,

$$U = \int_{\Omega} (f \bullet u) dx + \int_{\Gamma} (t \bullet u) ds : \text{ the strain energy of the} \quad 5-4$$

loaded structure with f body loads, t surface loads,
and u nodal displacements

$$V = \sum r_i^2 l_i \pi : \text{ the volume.} \quad 5-5$$

Particle swarm optimization was implemented to obtain a solution to the problem formulation above. The parameters for this optimization are summarized in Table 5-1. The identically sized strut configuration from section 5.1.1 was utilized as a ‘seed’ in one entry of the initial swarm configuration. This narrows the focus of the optimization

process, increasing convergence speed, without unduly limiting the potential design scope.

Table 5-1: Parameters for particle swarm optimization of first example problem

Maximum Number of Iterations	200
Error Goal	0.0001
Cognitive Acceleration	2.0
Social Acceleration	1.25
Neighborhood Acceleration	1.0
Initial Velocity Weight	0.4
Final Velocity Weight	0.95
Maximum Velocity Step	4.999

Since particle swarm optimization is a stochastic process, the optimization routine was run several times. As can be seen in Table 5-2 there was some degree of variation between the results of the multiple trials, as was expected. The best result, as can be seen in Figure 5-3, required 9605 seconds to complete, and achieved an objective function value of 2519.8. The optimization progression showed a fairly steady decrease in the objective function value throughout. There are a few large drops in the optimization function value, which is typical of particle swarm optimization. Since the process is stochastic, there are often abrupt changes in the progression of the objective function when a particular particle “discovers” a good solution.

To emulate the topology creation process employed by the unit-cell library method, struts falling in the lower 2.5% of the diameter range were removed from the structure. This resulted in an optimized structure, shown in Figure 5-4, with a volume of 1603.4mm^3 , strain energy of 5.006Nmm , and a maximum tip displacement at the top nodes furthest from the fixed end of the beam of 0.533mm . While this result is certainly better than the initial configuration, there are several unexpected aspects. Foremost of

these is the fact that the result is not symmetric across $y=5\text{mm}$, which would be expected since the problem statement is symmetric across this plane. This illustrates the random nature of particle swarm optimization. Even though it is better than some routines at avoiding local minima, this problem illustrates that it is still subject to focusing on *a* solution, rather than *the* solution. Figure 5-4 suggests that while particle swarm optimization was able to improve upon the initial solution, there exists a better solution that is most likely symmetric.

Table 5-2: Results of PSO for the First Example

Trial	F(x)	Volume (mm ³)	Max Displacement (mm)	Strain Energy (Nmm)	Time (sec)	Iterations
1	2519.8	1603.6	0.5326	5.0059	9359	201
2	3271.8	1602.6	0.5780	5.7133	9358	201
3	2519.8	1603.6	0.5326	5.0059	9640	201
4	3271.8	1602.6	0.578	5.7133	9620	201
5	2565.9	1601.0	0.5097	5.0633	9605	201
1*	N/A	1603.4	0.5327	5.0059	N/A	N/A

1* is the topologically reduced formulation of the first trial

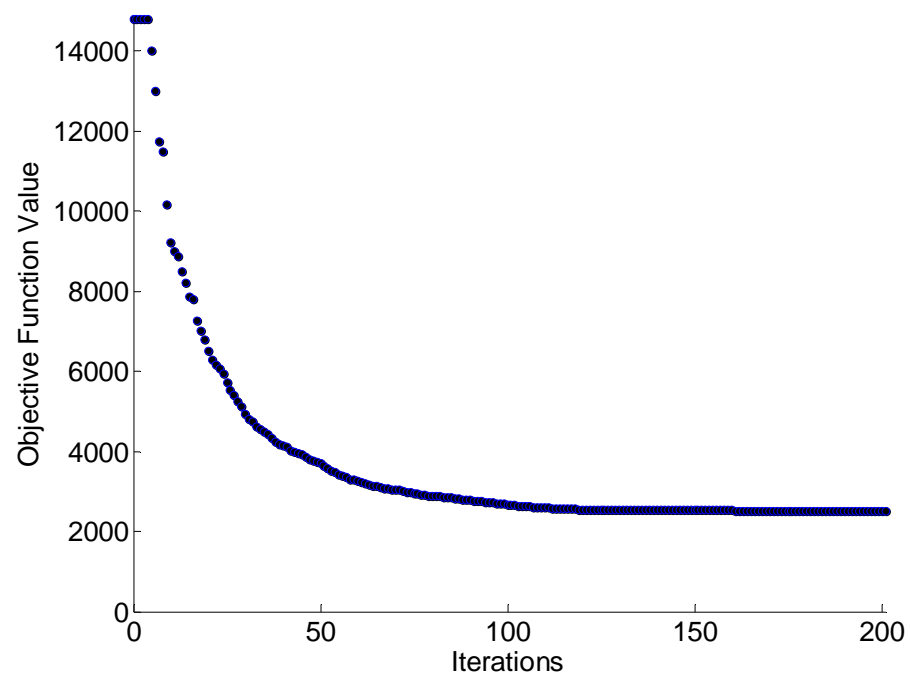


Figure 5-3: Iteration history for PSO optimization of first example problem

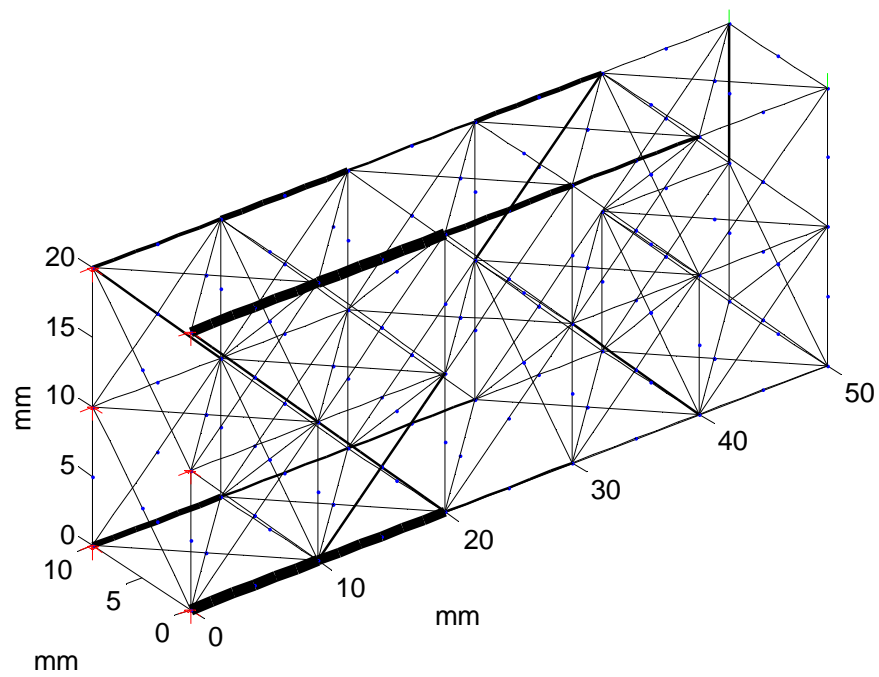


Figure 5-4: First example lattice structure after PSO optimization

5.1.3: Levenburg-Marquardt/Least Squares Minimization

The third design approach utilized was optimization of the lattice structure utilizing Levenburg-Marquardt/least squares minimization (LM/LSM). The problem formulation for LM/LSM, presented below, is similar to that presented in section 5.1.2 for particle swarm optimization.

Given: The problem statement from section 2.1

Find: The diameters of the 166 struts present in the model

Satisfy: No constraints

Minimize:

$$\min_D f(x) = w_U \times U^2 + (V - 1600)^2 \quad 5-6$$

Where: D are the truss strut diameters, $w_U=100$ is a weighting value, U is the strain energy V is the volume.

As a result of the problem formulation and the LM/LSM routine implemented, minimum and maximum diameter constraints could not be explicitly specified in the problem definition. The LM/LSM process did not tend towards instability, however, and the results fell within the 0.001-5mm range utilized for PSO. The other parameters for the LM/LSM procedure are summarized in Table 5-3.

Table 5-3: Parameters for least squares minimization of the first example problem

Termination tolerance on the function value	0.001
Termination tolerance on x	0.0001
Maximum Iterations	20
Initial Configuration	All struts 5mm

The LM/LSM routine ran for 2337 seconds to achieve an objective function value of 1127. Similar to the operation undertaken for the PSO solution, struts falling in the lower 2.5% of the diameter range were removed. This resulted in an optimized structure, shown in Figure 5-5, with a volume of 1601mm^3 , strain energy of 3.3537Nmm , and a maximum tip displacement of 0.3354mm . This structure reflects the symmetry present in the problem statement well, and shows intuitive placement of the lattice struts within the model.

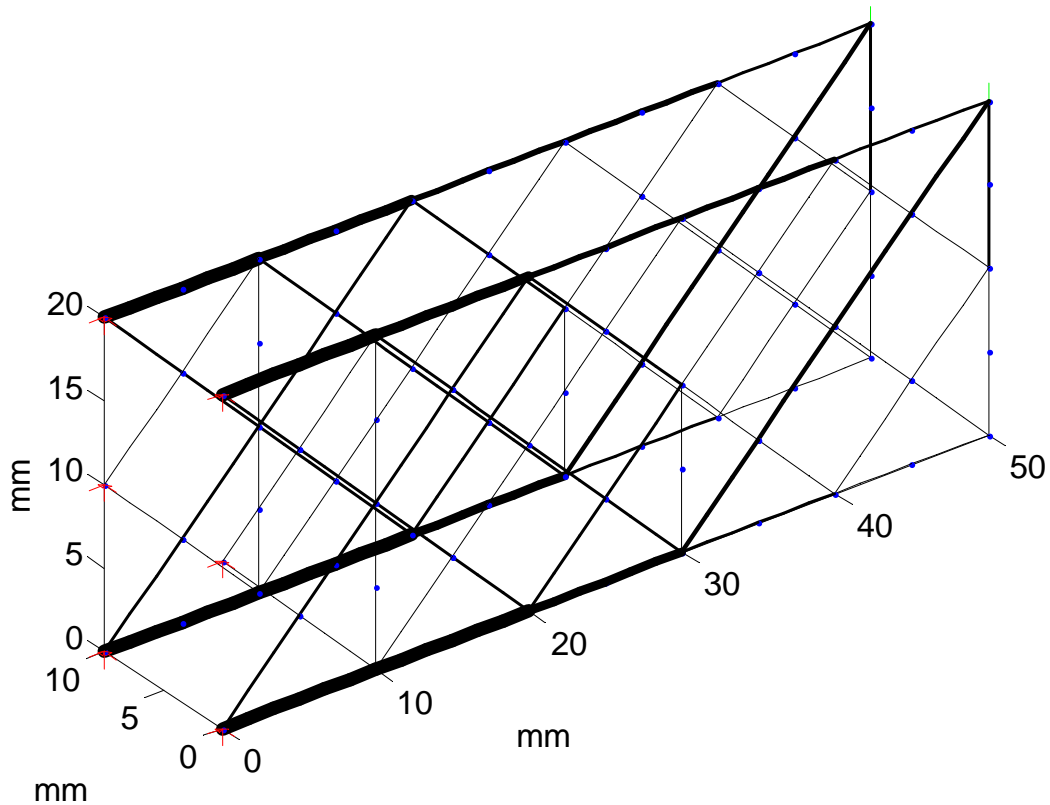


Figure 5-5: First example problem lattice structure derived using LM/LSM

5.1.4: Unit-Cell Library Approach

The final process utilized to design the lattice structure is the method developed in this work, which seeks to correlate stress values from a solid-body finite element analysis to entries from the unit-cell library. The first step in this process is the creation of a solid-body finite element analysis, shown in Figure 5-6, that is representative of the design problem. This analysis employed identical loading conditions and material constants as the design problem. Although the stresses, volumes, and displacements obtained through such an analysis do not directly correlate to the lattice structure, the relative stress distribution should apply to both geometries.

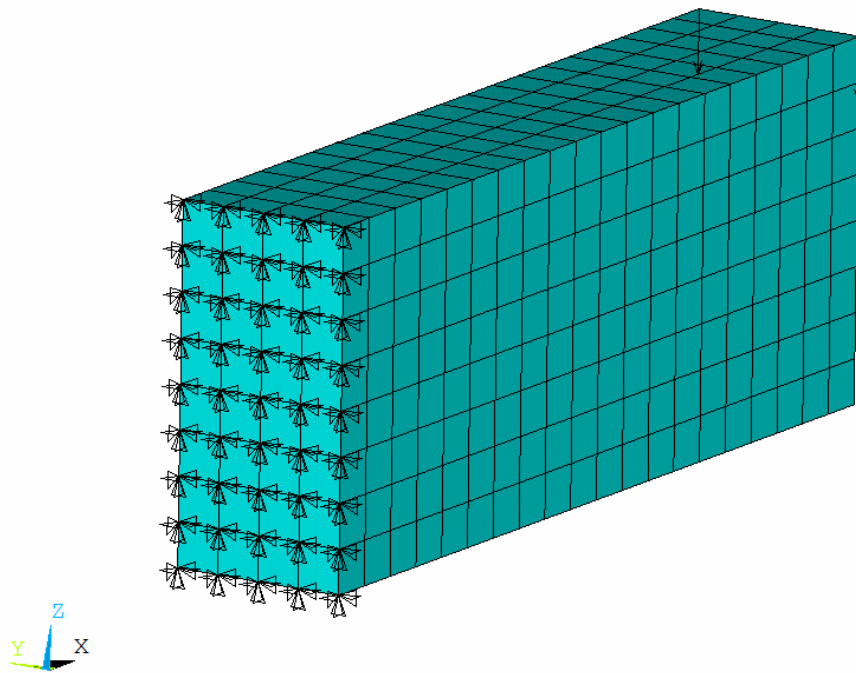
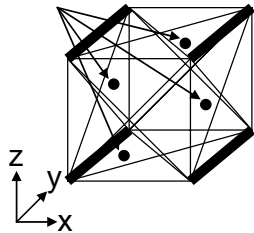
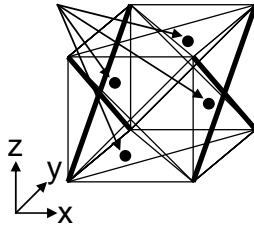


Figure 5-6: Solid-body finite element analysis of the cantilever beam example

The finite element model contains 945 nodes and 640 elements, and was intentionally constructed with a higher mesh density than the lattice structure in order to allow multiple finite element nodes to be averaged for each lattice unit-cell. After the

solid-body analysis was complete, the nodal stresses were normalized based on the largest stress present in the body. The normalized stress values associated with each unit-cell were averaged to determine scaling factors for cells throughout the model. This process is summarized in Table 5-4.

Table 5-4: Averaging and scaling of unit-cells

Scaled Unit Cell	Normalized FEA Nodes				Avg Scale	
<div>FEA NODES</div> 	1	2	3	4		
	σ_{xx}	0.08	0.1	0.07	0.11	0.09
	σ_{xx}	0.11	0.07	0.08	0.1	0.09
	σ_{yy}	0.88	0.9	0.89	0.92	0.90
	σ_{xy}	0.07	0.11	0.1	0.08	0.09
	σ_{xz}	0.07	0.08	0.11	0.1	0.09
	σ_{yz}	0.5	0.45	0.48	0.46	0.09
<div>FEA NODES</div> 	5	6	7	8		
	σ_{xx}	0.08	0.1	0.07	0.11	0.09
	σ_{xx}	0.11	0.07	0.08	0.1	0.09
	σ_{yy}	0.1	0.08	0.11	0.07	0.09
	σ_{xy}	0.07	0.11	0.1	0.08	0.09
	σ_{xz}	0.07	0.08	0.11	0.1	0.09
	σ_{yz}	0.5	0.45	0.48	0.46	0.47

Note that both unit cells in Table 5-4 contain larger struts in the primary direction of stress, but that the lower magnitude stress in the second unit-cell results in smaller maximum strut diameters in that cell.

Although the scales are automatically determined from the finite element analysis, the minimum, maximum, and cutoff diameters must be determined by the user. Section 4.6.1 determined that a preferred minimum/maximum diameter relationship is approximately 0.28, while an appropriate cutoff diameter is the lowest 2.5% of the minimum-maximum diameter range. Using these parameters to create a preferred lattice structure for this problem leads to a design with a maximum diameter of 2.5mm, a minimum diameter of 0.7mm, and a cutoff diameter of 0.745mm.

The resulting structure, shown in Figure 5-7, has a volume of 1615 mm³, strain energy of 5.547Nmm, and a maximum tip displacement of 0.5547mm. The design took 0.515 seconds to analyze in ANSYS, and 1.124 seconds to create topology. It shows intuitive strut placement, and reflects the inherent symmetry of the problem well. Of particular note, however, is the inclusion of large struts along X down the center of the structure. As shown by the LM/LSM solution, these struts are not needed for structural performance, but are included due to the forced symmetry of the unit-cells from the library. This affect could be reduced by increasing the density of the unit-cells in the structure, but that is not allowed as it would require alteration of the original problem statement.

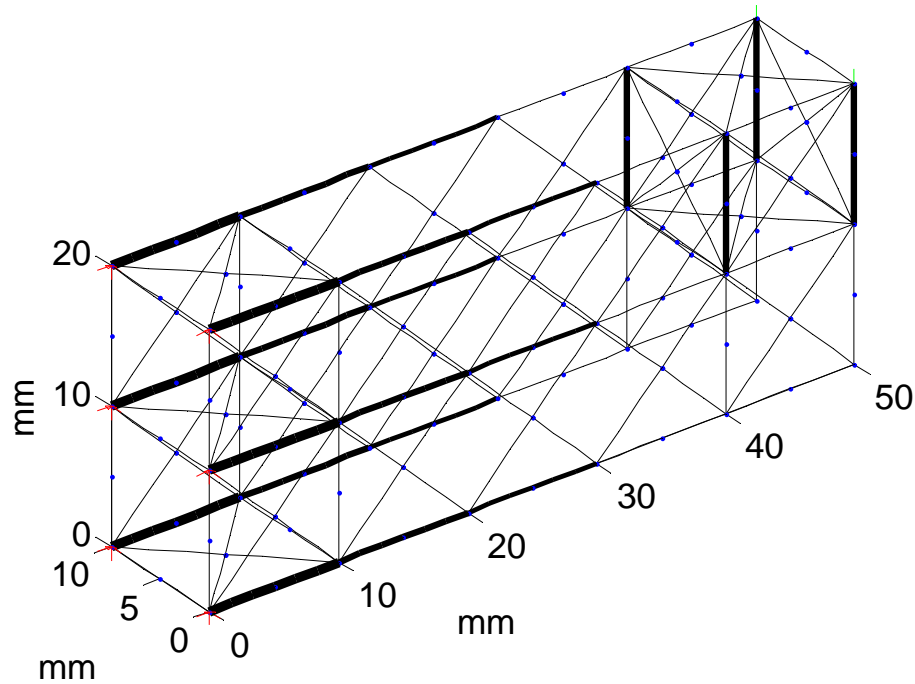


Figure 5-7: First example problem: unit-cell library and solid-body analysis

5.1.5: Summary of the First Example Problem

This example problem, the results of which are summarized in Table 5-5, illustrates several aspects of the various approaches for lattice structure design. It was expected that the unit-cell library/solid-body analysis approach would formulate a design faster than all but the identically sized method, and this expectation was borne out by the results of the example. It was also anticipated that the unit-cell library method would suffer noticeable degradation in structural performance compared to the optimization methods. This, too, was apparent in the results of the example problem, although the difference in the performance of the PSO method and the unit-cell library method was relatively small. For this particular example, the lower performance of the

unit-cell library method was largely due to the presence of significantly thickened struts along the center of the model. These thickened struts are a result of the basis of the unit-cell library method in finite element analysis. Assumptions made in FEA require that the elements be significantly smaller than the general part geometry. When applied to unit-cells nearly of an order as the part, as in this example, these assumptions start to break down. The struts down the center of the structure add significant amounts of weight to the structure, yet contribute little to the stiffness. This suggests that the unit-cell library method is best applied to designs which have significant differences in scale between the part and the lattice structure.

The possibility of a preferred minimum/maximum diameter ratio for the unit-cell library method is also alluring. If the specific ratio changes from one design to the next, its utility will be marginal. However, if it remains 28% across all designs, it will be possible to create preferable lattice structure designs for arbitrarily designated structure volumes or displacements.

Table 5-5: Summary of Cantilever Beam Example Results

Design Method	Volume (mm ³)	Maximum Tip Displacement (mm)	Strain Energy (Nmm)	Creation Time (s)
Identically Sized	1610	1.21	12.128	0.382
PSO	1603	0.5327	5.006	9359
LM/LSM	1601	0.3354	3.354	9283
Unit-Cell Library	1615	0.5547	5.547	1.639

5.2: Second Example Problem

This 2-dimensional example problem will compare the unit-cell library/solid-body analysis approach to the approach developed by Qi Xia and Yu Wang [37]. Xia and Wang have developed a gradient-based optimization approach for determining appropriate designs for both the geometry and topology of a truss structure that requires a fraction of the time necessary for particle swarm optimization or Levenburg-Marquardt/least squares minimization. In order to compare the results from the methods developed in this paper to those published by Xia and Wang, the initial condition from their first example problem will be duplicated. The results from their optimization of the topology will then be compared to results obtained using methods previously presented in this paper.

“The structure is loaded with a concentrated vertical force of $P=200\text{kN}$ at the center of the top edge and is supported on two hinges at the bottom-right corner and the bottom-left corner. The design domain is a rectangle of size $L=3\text{m}$, $H=1\text{m}$. The beams of the structure has[sic] a circular cross-section with the diameter $h_0=0.02\text{m}$the upper bound of the material volume [is] $\bar{V}=0.02\text{m}^3$. The penalty parameter in topology optimization is $p=2$, and the lower bound of topology variables $\underline{p}=0.04$. “[37]

The penalty parameter and bound for topology variables will be explained in the optimization sections to follow. Xia and Wang judged the performance of their structural

optimization problems by compliance, defined as $J = f^T u$ where f is the nodal force vector and u is the nodal displacement vector.

5.2.1: Identically Sized Truss Structure

In order to duplicate the starting configuration of the Xia and Wang paper, a truss structure, shown in Figure 5-8, was created with triangular shaped elements and strut diameters of 0.02 meters. Appropriate loading and boundary conditions were applied, and the results, Table 5-6, were compared to those of Xia and Wang. The model corresponded well, with the small amount of difference explained by the slight variation of the truss topology near the ends of the beam.

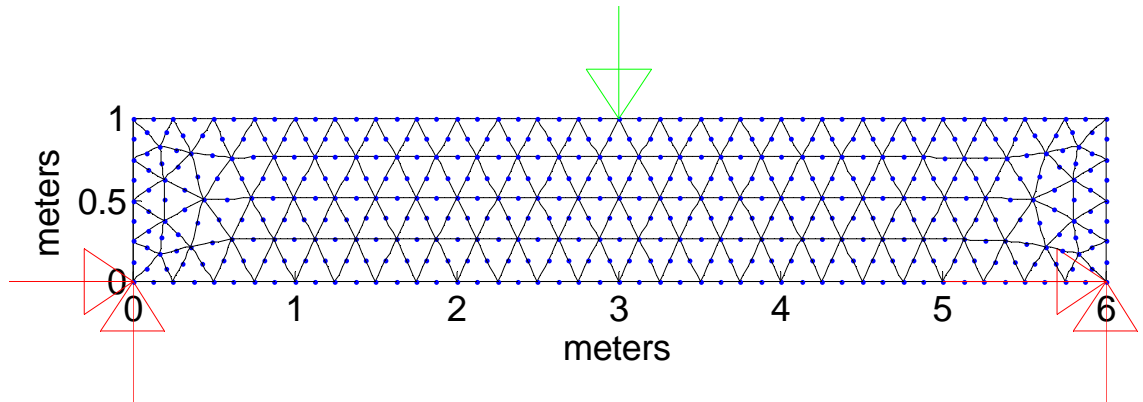


Figure 5-8: Identically Sized Triangular Truss Structure

Currently, the method developed to design truss structure based on solid-body analysis only allows for the inclusion of quadrilateral-based elements. Because of this, the original problem from Xia and Wang was re-meshed with quadrilateral unit-cells. An initial configuration was developed, Figure 5-9, with diameters equal to 0.02m. Since the initial configuration contained more struts than the triangular configuration, the volume and compliance characteristics did not correspond well to the triangular model, as can be

seen in Table 5-6. The designs that are based on this model, however, are subjected to the same volume constraints as the triangular-based designs.

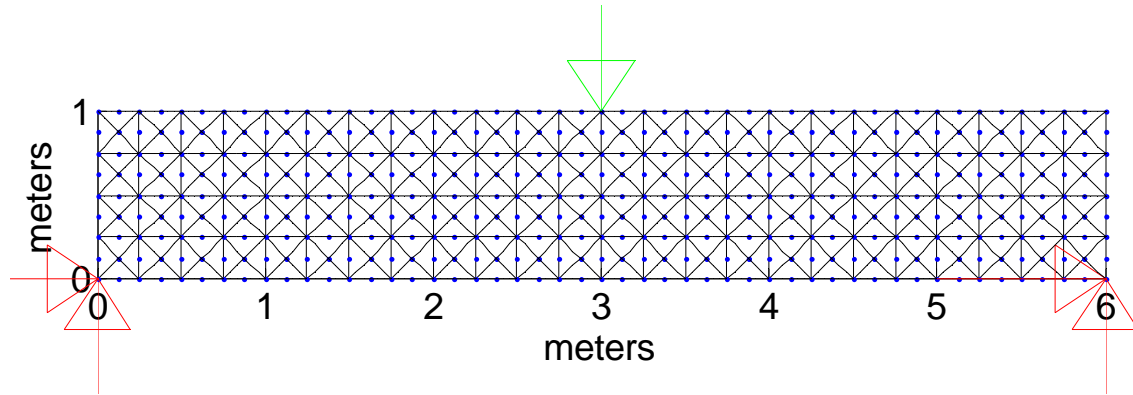


Figure 5-9: Identically Sized Quadrilateral Truss Structure

Table 5-6: Comparison of Starting Configurations

	Compliance (Nm)	Volume (m ³)
Xia and Wang	3325.96	0.0264
Triangular Model	3017.8	0.0271
Quadrilateral Model	2306.9	0.0386

5.2.2: PSO Optimization

As a first measure of comparison, the problem was optimized using particle swarm optimization. The problem formulation was based on the problem formulation used by Xia and Wang for their optimization procedure.

Given: The problem statement given for the second example

Find: ρ_e , a dummy size variable for each of the 328 struts

Satisfy:

$$0.04 \leq \rho_e \leq 1$$

5-7

Minimize:

$$\min_{\rho_e} f(x) = J + (w_v \times V) \quad 5-8$$

Where: $D_i = 0.02 \times \rho_e^2$ are the truss strut diameters, penalized towards the minimum or maximum diameter, and w_v is a volume penalty, with $w_v = 193000$ to scale the volume to a similar order of magnitude as the compliance.

Note that the original formulation of the objective function by Xia and Wang did not consider the structure volume. Instead, the volume was allowed to vary so long as it was below the defined maximum. The current PSO formulation does not allow for similar setting of arbitrary maximum parameters, so the volume was included in the objective function. The volume weighting value was set such that the resulting structure had a volume near the maximum allowed. A uniform diameter of 0.017m, a configuration with a volume below the maximum, was utilized as a seed in the initial population. Other control parameters are summarized in Table 5-7.

Table 5-7: Particle Swarm Optimization Parameters for Example Two

Swarm Size (# of variables)	100
Maximum Number of Iterations	50
Error Goal	0.0001
Cognitive Acceleration	2.0
Social Acceleration	1.25
Neighborhood Acceleration	1.0
Initial Velocity Weight	0.4
Final Velocity Weight	0.95
Maximum Velocity Step	4.999

The iteration history, shown in Figure 5-10, reveals that the majority of the optimization process takes place over only several iterations of the analysis, followed by a long period of limited improvement. This illustrates the stochastic nature of particle

swarm analysis, which can be greatly influenced by a particular member of the swarm that finds a “good” solution. Unfortunately, the point at which this occurs is unpredictable, as is the particular solution that will be identified. The analysis below did not satisfy any of the tolerance limits, and was ended by the maximum number of iterations parameter.

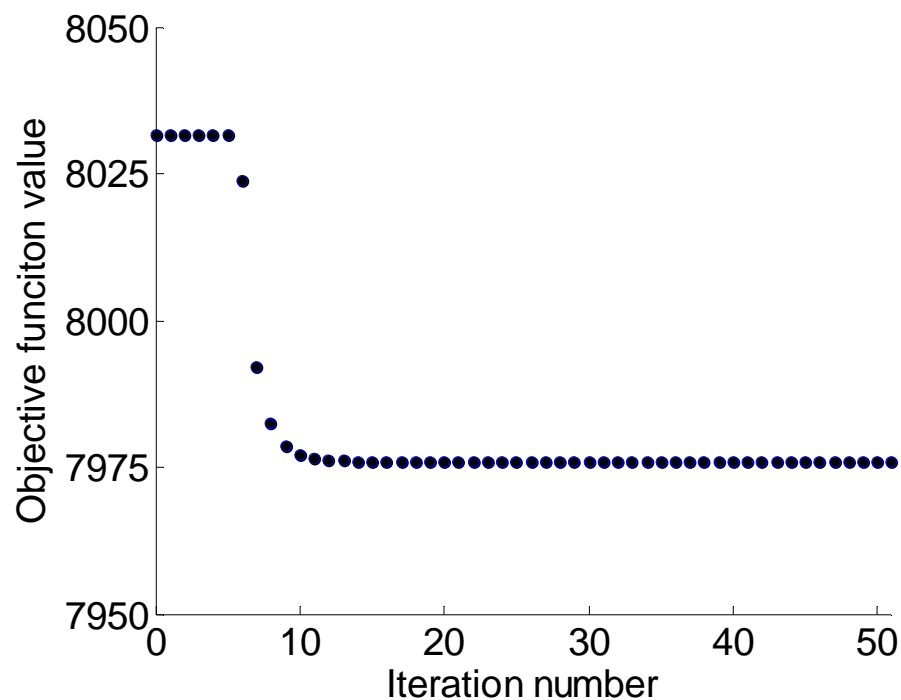


Figure 5-10: Optimization History of PSO for the Second Example Problem

Although the optimization resulted in a truss structure that was more satisfactory than the starting configuration, the final configuration was by no means definitive. Figure 5-11 illustrates several problems associated with this solution. For instance, the upper corners of the model are fairly thickened, even though this provides little support for the model. This illustrates the major limitation of particle swarm analysis; the solutions identified are usually better than the starting configuration, but are rarely the most

efficient design that could be produced. Increasing the swarm size during analysis alleviates this, but quickly proves impractical as the scale of problems increases. Pertinent results from the particle swarm analysis are summarized in Table 5-8.

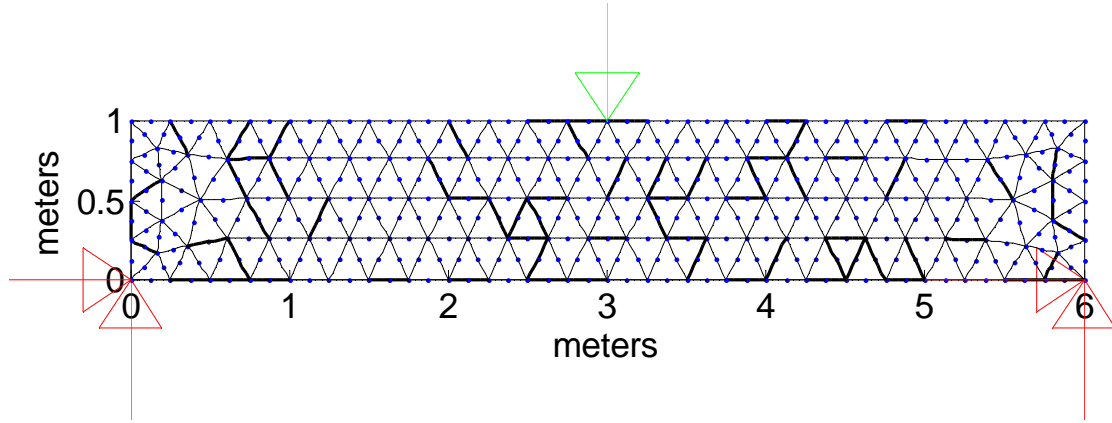


Figure 5-11: Example problem two after PSO optimization

Table 5-8: PSO results for the second example problem

Optimization Time (seconds)	4754
Function Calls	5200
Final Objective Function Value	7975.9
Iterations	51
Final Volume (m ³)	0.0196
Final Compliance (Nm)	4195

5.2.3: Levenburg-Marquardt/Least Squares Minimization

The problem was then optimized using Levenburg-Marquardt/least squares minimization. The problem statement for this process is as follows:

Given: The problem statement and ground structure for the second example

Find: ρ_e , a dummy size variable for each of the 328 struts

Satisfy: No specified constraints

Minimize:

$$\min_{\rho_e} f(x) = J^2 + w_v (V - 0.02)^2 + w_p P_d^2 \quad 5-9$$

Where: $D_i = 0.02 \times \rho_e^2$ are the truss strut diameters, penalized towards the minimum or maximum diameter,

$J = f^T u$ is the compliance,

$V = \sum r_k^2 l_k \pi$ is the truss structure volume, and

$P_d = \sum (D_k - 0.02)$ is a diameter penalty on all struts whose diameters are over 0.02m

$w_v = 10^{12}$ and $w_p = 10^8$

Note that, due to the nature of the least squares optimization routine, it was not possible to explicitly define the maximum volume and diameter constraints. Instead, the volume was included in the objective function, as was a penalty on any strut diameters over 0.02m. The presence of these variables in the objective function is solely to apply the constraints specified by Xia and Wang in the original problem. The other parameters for the LM/LSM procedure are summarized in Table 5-9.

Table 5-9: Parameters for least squares minimization of the first example problem

Termination tolerance on the function value	0.01
Termination tolerance on x	0.001
Maximum Iterations	20
Initial Configuration	$\rho_e = 0.8$

The Levenburg-Marquardt routine converged based on the maximum number of iterations, taking 6990 seconds and achieving an objective function value of 1.190×10^7 . The objective function history, shown in Figure 5-12, shows that there was a steady decrease of the objective function throughout optimization. The decreasing curvature

towards the end of the optimization implies that, although the maximum number of iterations stopped optimization, little decrease in the objective function would be realized by further iterations. The resulting structure is shown in Figure 5-13, and shows intuitive placement of the wider struts and reflects the symmetry of the problem well. The compliance of the resulting structure was 3432.2Nm, and the volume was 0.0201m³. These results are summarized in Table 5-10

Table 5-10: LM/LSM results for the second example problem

Optimization Time (seconds)	6990
Function Calls	6976
Final Objective Function Value	1.19x10 ⁷
Iterations	20
Final Volume (m ³)	0.0201
Final Compliance (Nm)	3432.2

While the Levenburg-Marquardt/least squares minimization required more time to complete than the particle swarm optimization procedure, the results obtained were significantly superior. Specifically, the Levenburg-Marquardt/least squares minimization procedure achieved a lower compliance value on the second iteration than the particle swarm optimization procedure achieved at the end of optimization. While the diameters of the structure were not within the allowable range until the fourth iteration, the volume was below the specified limit throughout the optimization process. This highlights the superiority of the LM/LSM process. While the optimization took longer, it was able to continue reducing the optimization function throughout the entire optimization.

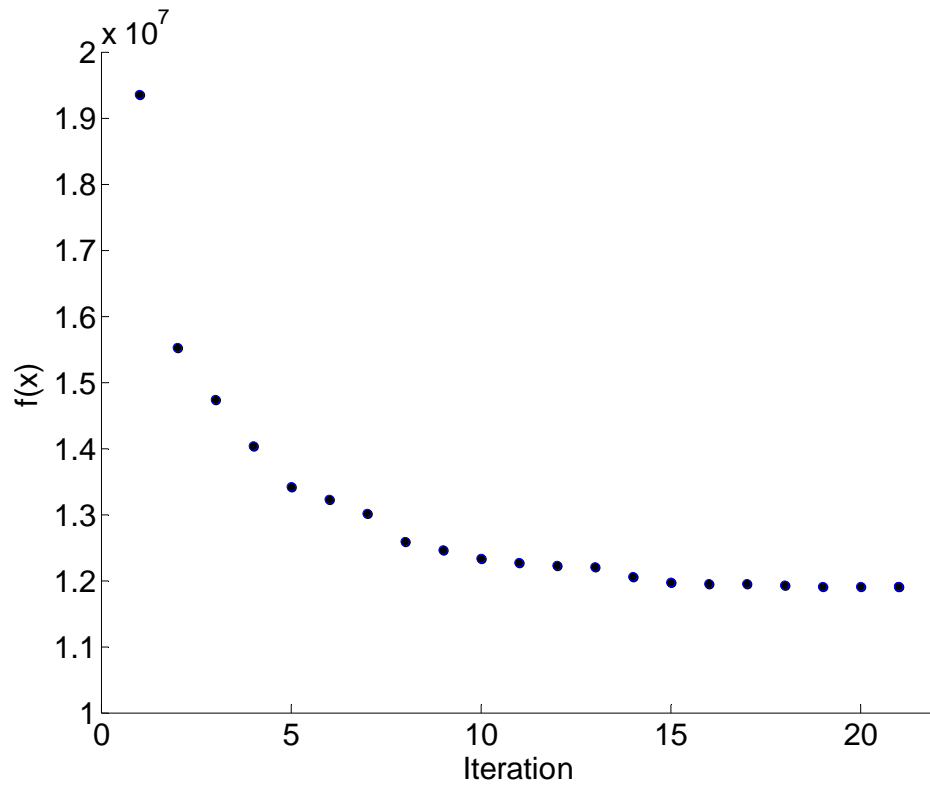


Figure 5-12: Objective Function History for LM/LSM of Second Example

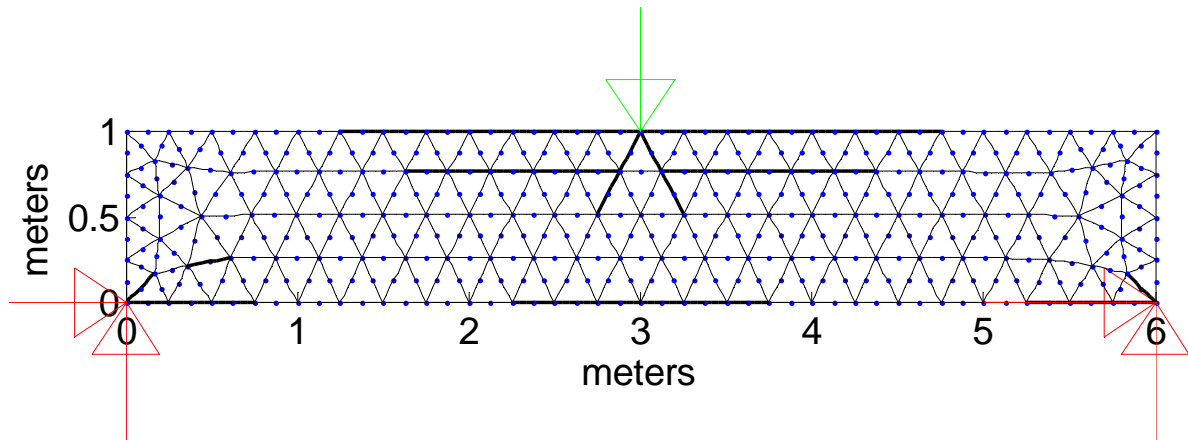


Figure 5-13: Result of LM/LSM of Second Example

5.2.4: Unit-Cell Library and Solid-Body Analysis

The process of developing a truss model utilizing the unit-cell library and solid-body analysis is identical to that undertaken during the first example problem. The problem was modeled as a solid plate in ANSYS, and identical loading and boundary conditions were applied. The stress distribution resulting from this analysis is shown in Figure 5-14.

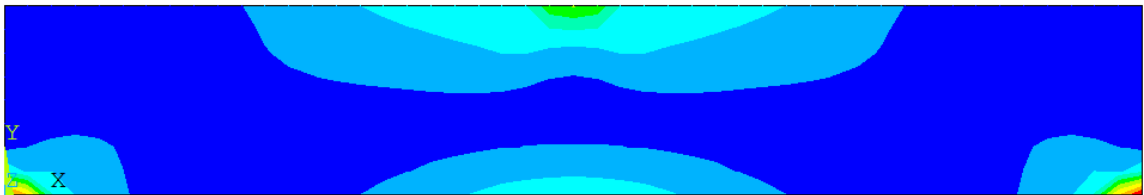


Figure 5-14: Ansys Stress Distribution in Second Example

The nodal and stress data were utilized to scale appropriate entries from the unit-cell library, in an identical manner as that undertaken for the first example problem. With a cutoff percentage of 2.5%, the resulting truss structure appears as that in Figure 5-15.

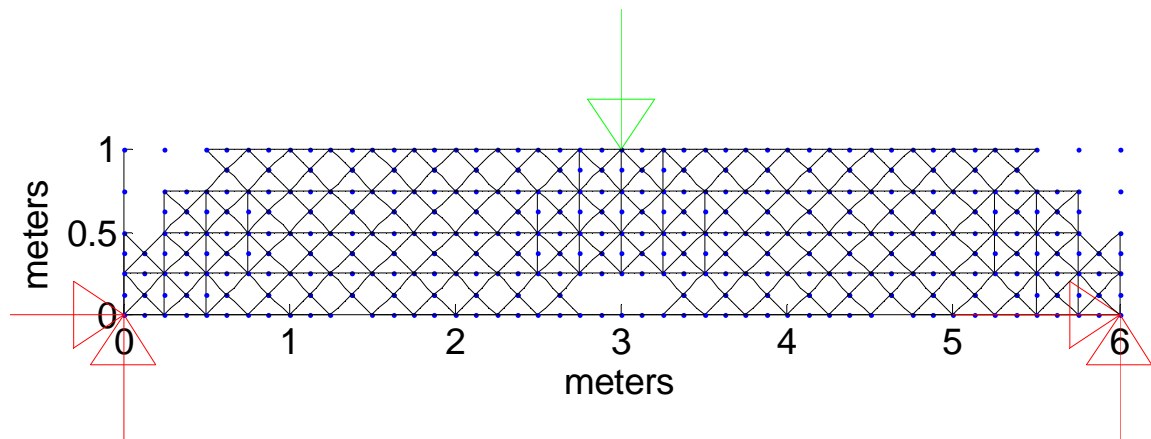


Figure 5-15: Truss Structure Specialized Using Solid-Body Analysis

Although the general topology is now developed, the minimum and maximum diameters have yet to be set. In section 4.6.1, Figure 4-7 illustrates that the least compliant structure for a given maximum diameter is that with the largest minimum diameter, so long as volume constraints are observed. Figure 4-8 suggests that an optimum minimum/maximum diameter ratio may be equal to or near 0.28. To corroborate these previous results, Figure 5-16 plots a range of maximum diameters with minimum diameters set to keep the structure volume constant at 0.02m^3 .

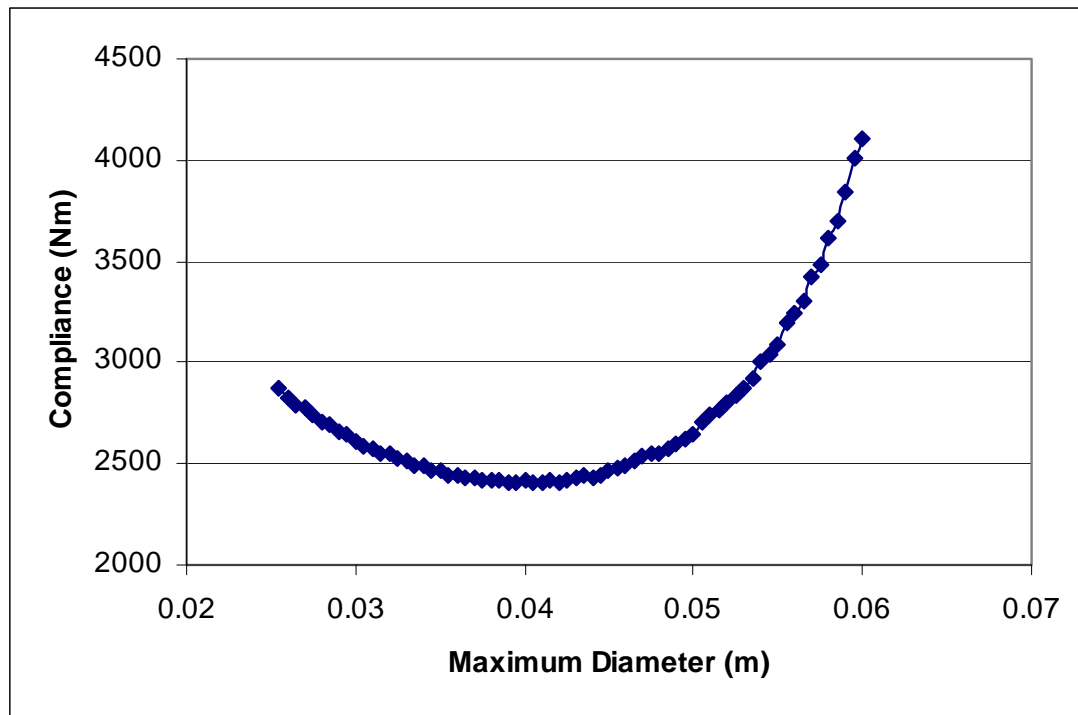


Figure 5-16: Plot of the Effect of Maximum Diameter on Compliance

The diameter pair with the lowest compliance, 0.01/0.039, corresponded to a ratio of 0.2564. While this differed from the previous results, there was only a 0.3% difference in compliance between the 0.2564 ratio and the diameter pair with a 0.28 ratio. This suggests that, so long as the diameter ratio is selected near 0.25-0.28, the resulting truss structures will be near the best performing configuration for a given structure.

Table 5-11: Results of Lattice Structure Created Using Solid-Body Analysis

Minimum Diameter (m)	0.01
Maximum Diameter (m)	0.039
Volume (m ³)	0.0201
Compliance (Nm)	2405.6
ANSYS Analysis (sec)	0.265
Lattice Creation (sec)	1.715

5.2.5: Summary of Second Example Problem

This section served to quantify the performance of the unit-cell library approach, as well as the optimization method developed by Xia and Wang. In this example, the results of which are summarized in Table 5-12, the unit-cell library method outperformed all other design methods in both structural performance and the time needed to develop a feasible design. As previously noted, the particle swarm optimization solution to this problem was particularly inefficient, and is of no particular use to the designer compared to the other solutions. Although the Xia and Wang method of optimization has dramatically reduced optimization times, the structure created with Levenburg-Marquardt/least squares minimization performed better, even though it required significantly more design time.

The time required to implement the unit-cell library remains an order of magnitude smaller than even the Xia and Wang method. The truss developed with the unit-cell library approach had more desirable structural characteristics, but it is difficult to compare the structural performance of the two design approaches. One reason that such a comparison is difficult is that the finite-element analysis utilized in this work is not optimized for efficiency. Since any optimization procedure involves multiple calls of the finite-element analysis routine, increases in analysis times are compounded throughout the optimization. Also, one design is based on a triangular unit-cell and the

other is based on a quadrilateral. They are essentially two different design problems. It would be more illustrative if an identical truss structure problem was formulated and analyzed by both methods, allowing direct comparisons to be drawn.

Table 5-12: Summary of Results for the Second Example Problem

Design Method	Volume (m ³)	Compliance (Nm)	Creation Time (s)
Identically Sized Tet	0.0271	3017.8	1.00
Identically Sized Quad	0.0386	2306.9	1.47
PSO	0.0196	4194.9	4,754
LM/LSM	0.0201	3432.2	6,990
Unit-Cell Library	0.0199	3400.0	1.70
Xia and Wang Initial Configuration	0.0264	3325.96	X
Xia and Wang Optimization Results	0.020	3595.19	34.13

5.3: Example Problem Three – A Complex Problem

The third example problem will serve as an example of the true utility of the unit-cell library lattice structure design approach. When lattice structures get suitably complex, the unit-cell library approach is not merely a potentially faster option among the analysis tools, but the only feasible design approach available to the designer. This example will show that the unit-cell library approach is able to create lattice structure designs for problems where other approaches are unable to do so.

The design problem is to create a saddle-shaped lattice structure. The surface of the skin is shown in Figure 5-17. This problem proves particularly challenging on two fronts: The geometric complexity makes defining deterministic unit-cells difficult, and the number of lattice struts inhibits efficient identification of preferred diameters.

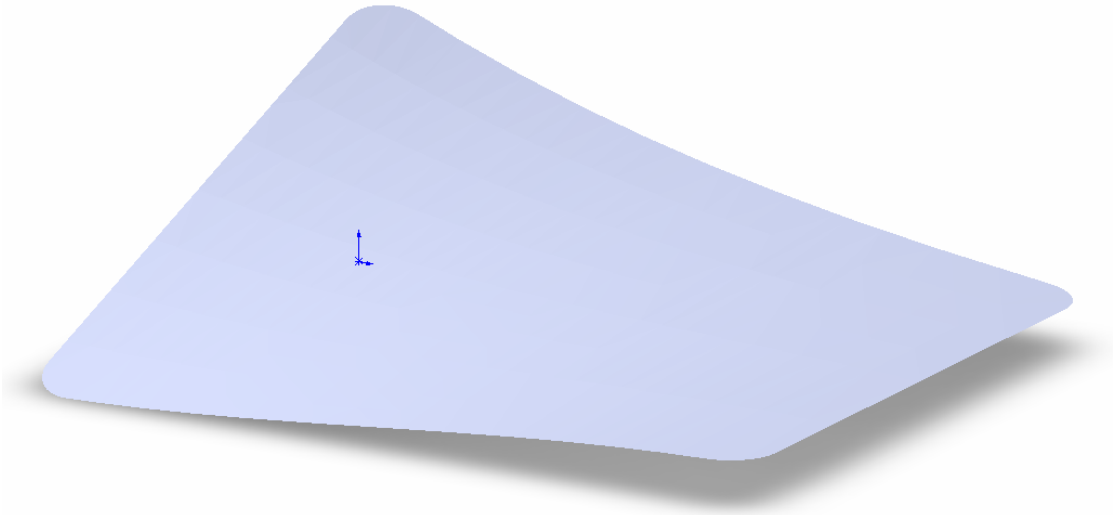


Figure 5-17: Saddle Skin Geometry

The design will consider a compressive force in the center of the structure, and performance will be gauged by the maximum displacement of any node within the lattice.

Other pertinent information for the design is as follows:

Dimensions: 130×100mm, 10mm thick lattice

Boundary Conditions: All edges fixed

Applied Load: 90N, distributed over an approximate 15mm by 15mm area

Desired Maximum Volume: 2700mm³

To define the locations of the lattice unit-cells, a method developed at Georgia Tech [14] was employed that offsets an arbitrary surface and places unit-cells between the original and the offset surface. This method is able to create deterministic unit-cells based on surfaces that are difficult or impossible to define analytically. As a baseline configuration, a lattice structure was defined with identically sized strut diameters, set equal to 1mm to create a lattice with a volume of 26976mm³. The resulting design, shown in Figure 5-18, was comprised of 4280 struts and had a maximum deflection of 0.82mm.

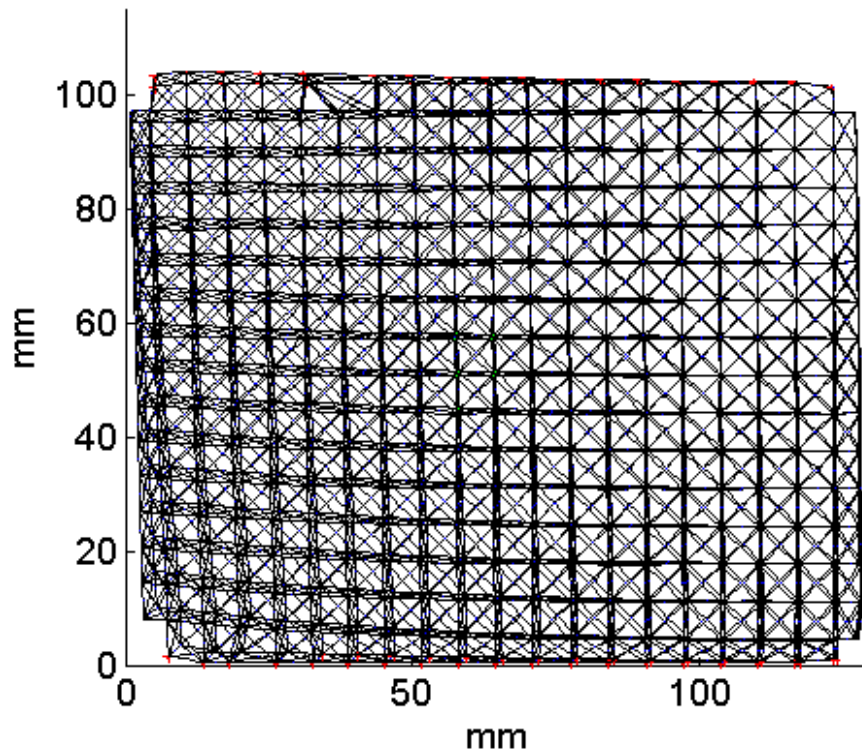


Figure 5-18: Saddle-Shaped, Identically Sized Lattice Structure

5.3.1: Discussion on Optimization

The truss structure above requires 1296 seconds for a single analysis. That is, for any individual design, it requires 1296 seconds to determine corresponding nodal displacements. As will be enumerated, this large analysis time proves prohibitive when attempting to determine lattice strut sizes using current optimization techniques.

Given the time required to analyze a structure, it is possible to approximate the time required to complete particle swarm optimization on that structure. In general terms, the two parameters that affect run time are the number of iterations, and swarm size. Since every iteration requires the structure to be analyzed once for each member of the swarm, the time required for PSO optimization can be roughly calculated by multiplying

the time required to analyze the model by the swarm size and expected number of iterations, as shown below.

$$\text{Optimization Time} = \text{Analysis Time} \times \text{Swarm Size} \times \# \text{ Iterations}$$

Typically, the swarm size is set to approximately one-third the number of variables in the problem. Thus, the swarm size of an optimization for the hatch cover example might be approximately 1000. At 1296 seconds per swarm member, each iteration of the PSO analysis would require 15 days to complete. The first example problem required 201 PSO iterations. If this problem were to require a similar number of iterations, it would take over 8 years to arrive at a solution. This would provide, quite obviously, little utility for practical design.

Estimating the time required to complete Levenburg-Marquardt/least squares minimization is more straightforward. The Levenburg-Marquardt/least squares minimization routine evaluates an iteration by individually adjusting each strut and analyzing the structure. The iteration is completed by adjusting all struts simultaneously and a final analysis. Thus, the number of times the structure is analyzed per iteration is equal to the number of struts present in the model, plus one. Since a single analysis of the structure takes 1296 seconds, and there are 4280 struts in the model, a single Levenburg-Marquardt iteration requires 64 days to complete. The first, fairly simple, example problem required twenty iterations to converge on a solution. If this is used as an indicator of the minimum number of iterations required for convergence, the saddle-shaped example problem would require 3 ½ years to converge. This too, is unacceptable for a practical design method.

5.3.2: Solid-Body Analysis and Unit-Cell Library Method

The method utilized to develop a lattice structure utilizing information from a solid-body analysis is identical to that undertaken in the first and second example problems. First, a solid model was created in ANSYS with similar boundary and loading conditions as the problem statement. Due to the model complexity, it was impractical to implement a mapped mesh of hexagonal elements, and a free-mesh comprised of tetrahedral elements was utilized instead. Although nodal locations differed slightly, loading conditions were set as close as possible to those in the lattice structure. The resulting model, shown in Figure 5-19, contained 3117 elements and 6460 nodes, whose stress information was passed to the lattice structure creation routines.

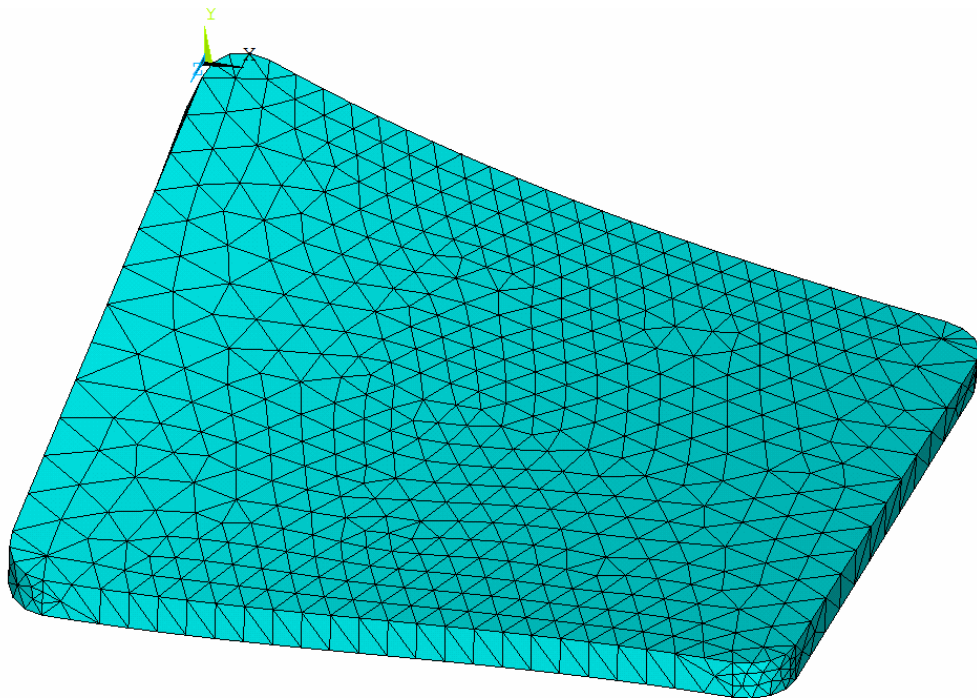


Figure 5-19: Ansys model of the Third Example Problem

The ANSYS node information and normalized stress data was used to scale entries from the unit cell library as they were implanted into the lattice design. The maximum diameter was chosen such that, when the previously determined relationships between the diameters were applied, the structure had the desired volume. The maximum diameter was set to 2.9mm, with the minimum diameter set at 30% of the maximum: 0.725mm. Struts with diameters in the lower 2.5% of the range were discarded. The resulting structure, shown in Figure 5-20, had a volume of 26982mm^3 and a maximum displacement of 0.68mm.

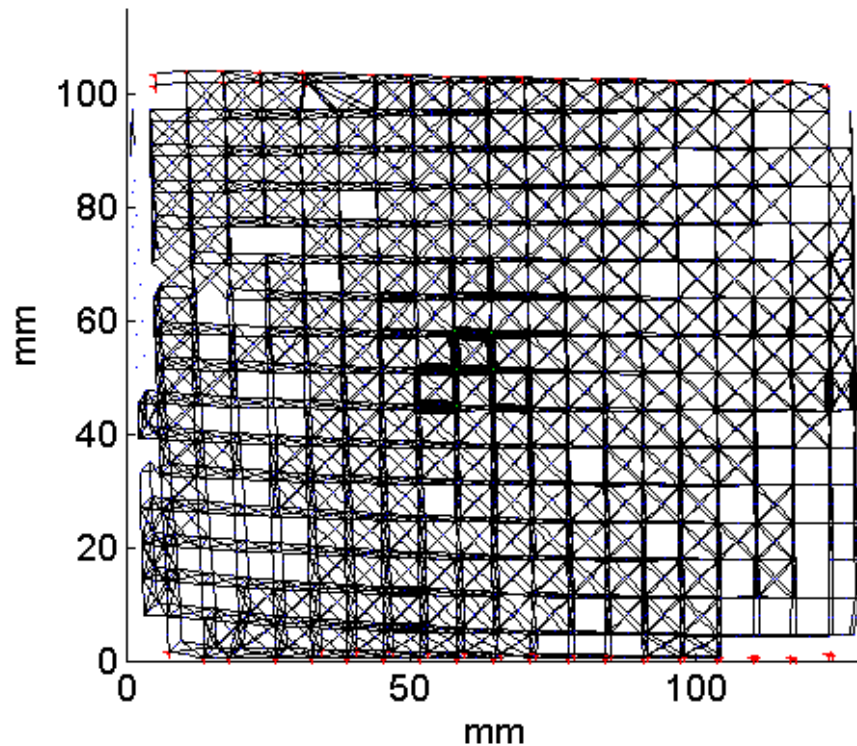


Figure 5-20: Specialized Lattice Structure for Saddle-Shaped Example

5.3.3: Summary of Third Example Problem

This example problem illustrates the benefits of the unit-cell library approach for large or complex lattice structures. Due to the number of elements present in the model,

and the resulting time needed to solve a finite-element model, current methods of optimization are impractical if not infeasible. Optimization times measured in weeks might make design challenging, but optimization times measured in months are prohibitive. This means that the two avenues of design available to the designer are a lattice structure comprised of identically sized elements, or a specialized structure created with the unit-cell method. In this example, implementing the unit-cell method provided a 16.8% reduction in displacement, while only increasing the design time by 49 seconds. The results from the problem are summarized in Table 5-13.

Table 5-13: Summary of Results from Third Example Problem

Design Method	Volume (mm ³)	Maximum Displacement (mm)	Creation Time	Analysis Time (s)
Identically Sized	26976	0.92	101(sec)	1296
PSO (estimated)	X	X	8 (years)	X
LM/LSM (estimated)	X	X	3 ½ (years)	X
Unit-Cell Library	26982	0.68	153 (sec)	902

5.4: Summary

The goal of this chapter was to provide evidence to support the research hypotheses by demonstrating that they satisfy several of the experimental tests. The specific experimental tests addressed in this chapter are numbers 2, 3, and 4, below:

- 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)
- 3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design

- 4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods.

Satisfying the second experimental test requires that the design method create lattice structures without the need for optimization routines. Throughout the example problems, lattice structures were created both with and without optimization. Those examples that utilized the unit-cell library method successfully created comparable lattice structures without implementation of optimization routines. This was accomplished through the inclusion of stress information from analysis of the structure geometry, which satisfied the second experimental test.

The remaining experimental test, and perhaps the most important, concerns the performance of the resulting lattice structure. In order for the method to prove useful, it must create designs with comparable structural characteristics as other methods currently available. It was expected that the structural performance of lattice structures would be slightly reduced when designed using the unit-cell library approach. This was borne out by the results of the first example problem. However, the second example problem suggests that, for designs of significant differences in scale between the part and lattice structure, the gap in difference in performance decreases. The results of particle swarm optimization, which were lackluster at best, suggest that as design problems get increasingly complex, it may become infeasible to identify appropriate solutions through the use of optimization. In these cases, the unit-cell library method seems to be able to outperform optimization methods in the time required to complete design as well as the performance of the resulting structure.

Chapter 6: Summary and Conclusions

The advent of rapid manufacturing has enabled the realization of countless products that have heretofore been infeasible. From customized clear braces, to jet fighter ducts and one-off dental implants, rapid manufacturing allows for increased design complexity and decreased manufacturing costs. The manufacturing capabilities of this process are so evolved, in fact, that they have surpassed the design capabilities of the designer. Meso-scale lattice structures can now be built that contain more lattice struts than it is reasonable to efficiently design. This work has attempted to create a method for designing such lattice structures that is efficient enough to allow for the design of large or complex problems. To this end, the following research questions and hypotheses were identified:

- Research Question #1: Can a method for designing deterministic meso-scale lattice structures be developed that is efficient enough to allow for the design of highly complex lattice structures?
- Research Question #2: If a method exists for designing lattice structures with reduced need for optimization, can such a method be implemented without significantly degrading performance of the final design compared to current design methods?
- Hypothesis #1: By utilizing a unit-truss library approach, in which individual truss configurations are chosen from a set of previously

optimized conditions, the majority of optimization can be removed from the design process and replaced with a process of selection of entire unit-cells. This reduces the computational requirements of design since each lattice strut need not be individually considered.

- Hypothesis #2: Solid body analysis of the geometry of a part can be used to guide the design process by matching individual unit-cells in a component with the corresponding stress conditions in the solid body. This information can then control the selection and sizing of components from the unit-cell library.

To gauge the effectiveness of the research hypotheses in satisfying the research questions, a series of research tests were developed:

- Tests for hypothesis #1
 - 1) Unit-cells can be specialized to exhibit certain characteristics through optimization
 - 2) Implementing specialized Unit-cells as a library eliminates rigorous global topological optimization (that is, optimization on the strut level)
- Tests for hypothesis #2
 - 3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design

- 4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods.

These tests were utilized throughout the work to judge the utility of the work completed, and its ability to satisfy the requirements of the research questions.

Chapter two provided a literature review of previous work that relates to the design of meso-scale lattice structures. This review included a discussion of stochastic and design cellular solids, and various methods of their analysis. Approaches for lattice structure optimization were identified, including Michell's analytical optimal trusses, homogenization, and the ground structure approach. Particle swarm optimization and Levenburg-Marquardt/least squares minimization, two methods of optimization, were presented and identified as being particularly well-suited to the problem of lattice structure optimization. Finally, a research gap was identified between the types of lattice structures that can be feasibly designed and those that can be analyzed or manufactured. It was determined that a method for lattice structure design that did not require the use of optimization would be desirable for the creation of complex lattice structures.

Chapter three discussed the development of a unit-cell library, presented the basis of the unit-cell approach, and how such an approach is beneficial to the design of lattice structures. After stipulating ways in which it would be beneficial to have a library of unit-cells specialized for certain purposes, a method for determining such unit-cells through rigorous optimization was presented. Several unit-cells were identified that perform best under compression/tension and shear stress conditions. Finally, the current unit-cell library was presented and briefly discussed.

Chapter four enumerated a method that has been developed for designing lattice structures without the need of optimization. This method selects and scales entries from the unit-cell library that have been specifically correlated to particular types of stress. Stress information for the model is generated from a finite-element analysis of the bounding geometry and loading conditions. The entries from the unit-cell library are then scaled based on the types and magnitude of stress present in the design.

Chapter five presented three example problems that illustrated the utility of the design method. The first example was a simple problem that was easily solved by methods of optimization. The second example was a more complex problem, which was difficult to solve through previous optimization methods. A new optimization procedure, which has only just recently appeared in the literature, was utilized to provide a more robust comparison. The last example problem was of a scale that previous methods of optimization were no longer feasible, and the only available options for design were an identically sized lattice or one designed using the new method. In all three examples, the new design method produced designs that were dramatically faster to create, and had comparable, although slightly reduced, structural performance. This provided strong support for the utility of this new design process.

6.1: Conclusions

After devising the tests to be used when validating the research hypotheses, a plan, shown in Table 6-1 was devised that identified which tests individual chapters would address. Throughout the thesis, this plan has dictated the structure and content of the work presented, as each chapter strove to identify the success or failure of the

hypotheses through analyzing the research tests. This section will draw final conclusions on the ability of the hypotheses to satisfy the research questions, based on the results of the four previously identified tests.

Table 6-1: Hypotheses Verification Outline

<div>Research Test</div> <div>Chapter</div>	1) Unit-cells can be specialized to exhibit certain characteristics through optimization	2) Implementing specialized Unit-cells as a library eliminates global topological optimization	3) Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design	4) Parts designed with this method do not undergo a significant degradation in performance compared to existing design methods
Development of the Unit-Cell Library (Chapter 3)	X	X		
Correlation to solid-body analysis and design method (Chapter 4)			X	
Example Problems (Chapter 5)		X	X	X

6.1.1: Specialized Unit-cells

The first research test was whether or not “unit-cell can be specialized to exhibit certain characteristics through optimization.” Chapter three, through the development of a unit-cell library comprised of such specialized unit-cell, provided strong evidence that such specialization can be accomplished. It provided a method for defining specialized unit-cells, and applied the method to the creation of cells specialized to perform best under compression/tension or shear. These specialized cells are shown in Figure 3-2. The development of these unit-cells, and their successful implementation to design lattice

structures, leads to the conclusion that it is indeed possible to create unit-cells that are specialized for certain applications or characteristics.

6.1.2: Strut-Level Optimization

The second research test was if “implementing specialized unit-cells as a library eliminates rigorous global topological optimization.” In the third chapter, it was stipulated that this could be accomplished through a selection-type optimization by using the unit-cells as optimization variables. Chapters four and five went a step beyond this by first explaining, and then demonstrating, a design method that completed the entire process of design without the need for any optimization. This was accomplished by using information from a finite-element analysis of the bounding design geometry and loads to guide the selection and scaling of unit-cells from the library. The successful completion of this process in chapter five conclusively proved that the use of the unit-cell library allows for design without the need of strut-level optimization.

6.1.3: Solid Body Analysis

The third research test was whether or not “Solid body analysis can be utilized to select and size unit-cells from the library during lattice structure design.” The fourth chapter exclusively addressed this research question by detailing a design method whose sole aim was to correlate the information from a solid body analysis to lattice structure design. The six library unit-cells that had been specialized for certain stress conditions were scaled based on the average stress present in the solid body analysis at each unit-cell. The entire model thus consisted of many unit-cells, each filled with the scaled entries from the unit-cell library. This process was further tested in chapter five, where it

was applied to actual lattice-structure design problems. The successful implementation of this method for design problems leads to the conclusion that, with correct correlation between the unit-cell library and solid body analysis, the solid body analysis can be used to select and size unit-cells from a unit-cell library during the design process.

6.1.4: Performance

The fourth, and final, research test stipulated that “parts designed with [the] method do not undergo a significant degradation in performance compared to existing design methods.” Investigation of this research test was conducted in chapter five, which compared the structural performance of structures designed using various design methods. A slight degradation of performance was expected, and realized in the problems. Although the unit-cell library method does not perform nearly as well as the Levenburg-Marquardt/least squares minimization optimization approach, it was on-par with the particle swarm optimization method. The results of the second and third example problems suggest that the value of the unit-cell library method lies in the creation of highly complex lattice structures that are impractical to design through optimization. This conclusion is in line with the stated goal of the design method.

6.1.5: Conclusions

Based on the positive results for the four research tests, the two hypotheses satisfactorily answer the research questions. This allows the following conclusions to be drawn:

Conclusion #1: A method for designing deterministic meso-scale lattice structures can be developed that is efficient enough to allow for the design of highly complex lattice structures by utilizing a unit-truss library approach, in which individual truss configurations are chosen from a set of previously optimized conditions. This removes the majority of optimization from the design process, and replaced with a process of selection of entire unit-cells. This reduces the computational requirements of design since each lattice strut need not be individually considered.

Conclusion #2: Such a method for designing lattice structures can be implemented without significantly degrading performance of the final design compared to current design methods by using solid body analysis of the bounding geometry of the part to guide the design process. This can be accomplished by correlating individual unit-cells in a component with corresponding stress conditions in the solid body. This allows the stress information from the solid body analysis to control the selection and sizing of components from the unit-cell library as they are placed in the lattice structure.

6.2: Contributions

The major contribution of this work is the concept of the unit-cell library and its implementation in lattice structure design. The unit-cell library and implementation for

lattice structure design are presented in chapters three and four, respectively. Several aspects of lattice structure design were furthered in the process of developing these processes, and some of these contributions are below.

6.2.1: Unit-Cell Library

While previous work has utilized unit-cells, and tailored unit-cells for particular applications, the concept of a library of different unit-cells that can be referenced for a single design is novel. Even if the unit-cell library is not used in conjunction with the design method outlined in this work, it can be implemented in such a way that the designer does not need to identify the correct topology and geometry for a design, but only the correct *type* of topology and geometry. Such an implementation would be similar to a gear or bearing reference. If enough types of unit-cells have been defined and well studied they can be implemented with relatively little effort on the part of the designer.

Paramount to the implementation of a unit-cell library is the ability to create specialized unit-cells with which to populate the library. Section 3.4 provided a framework for accomplishing this process of specialization. While the specific example was the creation of 8 node unit-cells undergoing specified types of stress, the framework provided could easily extend into other structural applications, other domains such as thermal or vibrations, or even multiple-domain problems. Such a framework allows for the extension of the unit-cell library when the current entries are not sufficient for the purposes of the component being designed.

6.2.2: Optimization-Free Design

In lattice structure design, the largest inhibitor to streamlined design has been the need for optimization to obtain results with satisfactory performance. A gap between modeling/analysis capabilities and optimization capabilities means that complex structures can be created and manufacture, but not efficiently designed. The method of design presented in this work overcomes this barrier by accomplishing design without the need for optimization. This removal of optimization from the design process means that designs are limited by only the ability to analytically represent and manufacture the structure. While structures created using the new design method suffer degradation in performance when compared to optimized structures, the value of the method lies in its application to problems whose scale renders optimization impractical. For the current set of methods available for the design of such complex structures, which exclude optimization techniques, this work has contributed a design method that is both faster and produces superior results.

6.3: Future Work

Several areas have been identified in which further results could be realized through future work.

6.3.1: Streamlining

The current implementation of the design process is very labor intensive for the designer. Design parameters such as bounding geometry or nodal stress data must be manually defined or manipulated by the designer. The entire design process could be

streamlined to require less work from the user. In the ideal case, this would require the designer to only specify the bounding geometry, loading conditions, and desired design requirements such as displacement or volume constraints. Admittedly, much of this process concerns software and user interface design rather than structural performance concerns. However, such work would allow for the broad implementation of meso-scale lattice structures outside of the laboratory.

An additional avenue of streamlining is the potential to combine the unit-cell library design method with current optimization techniques. By utilizing a potential solution from the unit-cell library method as the basis for future optimization, the time required for optimization may be greatly reduced. This might provide a useful balance between the speed of the unit-cell library approach and the structural performance provided by optimization techniques. Since such a process would require optimization, the complexity of applicable problems would still be limited, but this requirement might be somewhat mitigated. This would improve results for structures which currently lie on the boundary of feasible optimization problems. By reducing the time required for design, structures which are currently almost impractical to optimize may become realizable.

6.3.2: Solid Body/Unit-Cell Library Correlation

Currently, entries exist in the library that correlate to axial stress in xx , yy , and zz , and shear stress in xy , xz , yz . While these entries correlate well to the stress data available from finite element analysis, their use implies a basic assumption that the unit-cell is significantly smaller than the scale of the part. This assumption may not necessarily be valid for all problems. For such problems, it might be beneficial to include entries in the unit-cell library that include such higher-order effects like bending or

torsion. These could then be paired with a solid body analysis conducted with higher-order finite elements that are able to provide local bending or torsion data. While such work would not entail a fundamental shift in the theory of the design process, it may allow for the design of better performing structures without significant increases in the time required for such design.

6.4: Closure

With the increasing prevalence of rapid manufacturing in industry, designers are continuously pushing the boundary of feasible designs. The products they create solve problems in novel or unexpected ways because they need not consider many of the constraints of manufacturing that have traditionally limited design. Increasingly, this is resulting in designs that are manufacturable, but beyond the scope of human design. In the case of meso-scale lattice structure, it is fairly trivial to propose designs that are even beyond the abilities of current automated design aids. If the true utility of rapid manufacturing is to be realized, the tools used during the design process must be adapted at the same speed as the products they are implemented to create. This work has advanced the capabilities of one small portion of this broad array of design tools and theories, but by doing so lays the groundwork for future advances and developments that will enable future designs to far surpass those that can be created today. In the future, design will be limited not by the complexity or scale of the design, but by the scope of the designer's imagination.

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